# The Home Math Environment and Math Achievement: A Meta-Analysis

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**Abstract**

Mathematical thinking is in high demand in the global market, but approximately six percent of school-age children across the globe experience math difficulties (Shalev, et al., 2000). The home math environment (HME), which includes all math-related activities, attitudes, beliefs, expectations, and utterances in the home, may be associated with children's math development. In order to examine the relation between the HME and children's math abilities, a preregistered meta-analysis was conducted to estimate the average weighted correlation coefficient (*r*) between the HME and children's math achievement and how potential moderators (i.e., assessment, study, and sample features) might contribute to study heterogeneity. A multilevel correlated effects model using 631 effect sizes from 64 quantitative studies comprised of 68 independent samples found a positive, statistically significant average weighted correlation of $r = .13$ ($SE = .02$, $p < .001$). Our combined sensitivity analyses showed that the present findings were robust, and that the sample of studies has evidential value. A number of assessment, study, and sample characteristics contributed to study heterogeneity, showing that no single feature of HME research was driving the large between-study differences found for the association between the HME and children's math achievement. These findings indicate that children's environments and interactions related to their learning are supported in the specific context of math learning. Our results also show that the HME represents a setting in which children learn about math through social interactions with their caregivers (Vygotsky, 1978), and what they learn depends on the influence of many levels of environmental input (Bronfenbrenner, 1979) and the specificity of input children receive (Bornstein, 2002).

**Public Significance Statement:** The findings of this meta-analysis suggest that children's home math environments (e.g., parent-child math interactions) are positively associated with children's math achievement. To promote children's math skills, it may be beneficial to support parents in providing positive home math experiences for their children.

Keywords: home math environment; math; meta-analysis

**The Home Math Environment and Math Achievement: A Meta-Analysis**

Mathematical thinking is in high demand worldwide, but across the globe, approximately six percent of school-age children experience math difficulties (Shalev et al., 2000). In an age when mathematical thinking has become integral to sustaining a competitive advantage in the global market, a growing body of evidence that early math skills predict later math achievement (Lyons et al., 2014). Given that math achievement deficits already exist at the onset of formal schooling (Aunola et al., 2004), school-based instructional efforts to improve math outcomes are likely not enough. Alongside research that has found a strong association, even beyond the effects of social class, between general home learning activities and student achievement (Bus et al., 1995; Kellaghan et al., 1993), evidence also shows that children's early math knowledge develops when they have opportunities to engage with and talk about math in a playful, low-stakes manner (Cohrssen et al., 2014).

The home math environment (HME), which encompasses all math-related interactions among parents and children in the home, including informal board game playing, parents' expressions of their math-related attitudes, beliefs, and expectations, using words that compare magnitudes (e.g., more, less), and other math-related exchanges and utterances, may provide a promising avenue for the development of children's early math skills before school entry. It stands to reason that variations in home math experiences are partially driving the large differences in children's early math skills prior to formal schooling (Aunola et al., 2004; Evans & Shaw, 2008; Huntsinger et al., 2016; Senéchal & LeFevre, 2002). However, the role of the HME in children's math achievement remains unclear, with reported correlations between the HME and children's math achievement ranging from small to large and positive to negative. Although previous meta-analyses have considered parents' involvement in their children's academic skills, these have been focused on general academics (i.e., not math-specific; Castro et al., 2015) or have focused only on children of a specific age (e.g., Dunst et al., 2017). Given this evidence, we conducted the present preregistered meta-analysis of the correlation between the HME and children's math achievement to evaluate the role of home-based math-related interactions in children's math outcomes.

## Theoretical Basis of the Link between the HME and Children's Math Achievement

The HME represents a setting in which children learn about math through social interactions with their caregivers (Vygotsky, 1978), and what they learn depends on the influence of many levels of environmental input (Bronfenbrenner, 1979) and the specificity of input children receive (Bornstein, 2002).

### *Social Interactions*

According to Vygotsky's (1978) sociocultural learning theory, children's cognitive abilities develop through social interactions with more experienced partners, pushing children toward an upper boundary of ability that they could not reach on their own. Within the HME, these social interactions may include parent-child math-related activities and utterances as well as the socialization of math attitudes through talking about feelings toward math and expectations for children's achievement in math. Importantly, these math-related interactions may be associated with a level of math learning that surpasses what children could achieve independently. In fact, these social learning opportunities within the HME may be partially driving the differences in math ability found in young children prior to formal schooling (Aunola et al., 2004). Accordingly, the present meta-analysis directly investigated how math-related social interactions and parent attitudes and expectations toward math within the HME were associated with children's math achievement.

***Environmental Moderators***

Bronfenbrenner's (1979) ecological systems theory proposes that interactions between children and five layers of environments, ranging from the most proximal—like the home environment (i.e., microsystem)—to the most distal—like the age or grade at which children experience certain inputs (i.e., chronosystem)—interact to influence the development of children's academic skills. The role of the HME within the ecological systems theory can be seen at all layers of the environment. Within the microsystem, which captures the direct environmental interaction with the child, parents directly influence their children's math skill development through the provision of math-related activities, attitudes, utterances, and resources within the home (i.e., the HME), all of which are examples of proximal processes. The mesosystem captures connections between microsystems, like the home and school. An example of a mesosystem influence captured by the HME would be parents holding high expectations for their children's math achievement and consequently, creating strong ties between home and school by working on school-related math skills at home. The exosystem captures connections between caregivers and outside environments that are indirectly connected to children, like the interaction between parents and their workplaces. For example, if a single mother works multiple jobs that require long hours to support her family, she is less likely to have time to provide enriching math-related learning experiences or discussions at home with her child. A potential proxy for this mesosystem influence that may impact the HME is socioeconomic status (SES). The macrosystem captures the overarching economic and social policies in place in a child's environment, as well as the influence of social norms and culture on the microsystem. For example, some cultures are more likely to emphasize parent involvement in academic achievement and set high standards for their children's school outcomes (Huntsinger et al., 2016), which translates into an increased emphasis maximizing children's math achievement, potentially through the HME. Finally, the chronosystem represents the influence of time and timing among all the layers of the child's ecological system. For example, the kinds of math activities that are developmentally appropriate during the preschool years may differ from the activities that are suited for an elementary school student. To account for these five layers of influence operating within the HME we conducted a series of moderator analyses, which enabled us to investigate how these ecological influences may influence the magnitude of the relation between the HME and children's math achievement.

***Specificity***

According to Bornstein's (2002) Specificity Principle, the nature and timing of children's math-related experiences within the HME influence how their math-related characteristics develop. The specificity principle posits that the development of children's domain-specific skills requires domain-specific inputs in a developmentally appropriate manner in terms of timing and difficulty (Bornstein, 2002). Thus, parent attitudes, expectations, utterances, and/or activities should be math-specific in order to influence children's math development. Furthermore, within the math domain, the specificity principle posits that children will only develop specific math skills if those specific skills are targeted. For example, if parents want their children to know how to add and measure, they have to work on and talk about adding and measuring directly. In order to examine the role of specificity in the HME-math achievement link we investigated the differing relations between how the HME is defined and different math assessment methods and how the age and/or grade or SES of the sample may have impacted the magnitude of the correlation between different HME inputs and math outcomes.

There are various theoretical reasons why the HME might be important for children's math skill development, and there are different ways we can conceptualize the HME itself, whether it is math-related activities, attitudes, beliefs, expectations, or talk. By using our combined study sample and meta-analytic techniques we were able to empirically determine some of the theoretical mechanisms at work in the HME-math achievement relation and how it was affected by nuances in measurement and sample characteristics.

**Home Math Environment Definitions and Moderators**

The findings on the association between the HME and child math achievement are inconsistent (e.g., Blevins-Knabe, 2000; Ciping et al., 2015; Huntsinger et al., 2016). Some studies have found a positive and significant correlation between the HME and math achievement in children ranging from pre-school to elementary school (e.g., Dearing et al., 2012; Manolitsis et al., 2013; Niklas et al., 2016a; Skwarchuk et al., 2014). Conversely, negative or non-significant associations have been reported in multiple studies (e.g., Ciping et al., 2015; Huntsinger et al., 2016; Missall et al., 2015). Given that theoretically there is a wide array of home-based factors potentially encompassed by the HME, and that the HME has been defined differently among different research areas and studies, these mixed findings are not surprising. A common thread among the many conceptualizations of the HME is the emphasis on parent[1] involvement with math (e.g., Niklas & Schneider, 2016), but there is no consensus on the specific components that should be included to capture this parent involvement. Overall, research examining the role of math-specific home-based learning in children's math achievement has ranged from single-factor definitions of the HME to a wide variety of multi-component definitions (e.g., Ciping et al., 2015; del Rio et al., 2017; LeFevre et al., 2009).

[1] Note: the terms "parent" or "parents" are used throughout the manuscript to refer to any primary caregiver(s) in the home.

Beyond how best to define the HME, research over the past few years has also been aimed at parsing the many variables related to HME measurement, math assessment, and study and sample characteristics that may be driving the differences in HME-math achievement associations found between studies. This includes varying the instrument used to measure the HME by including either observational or report-based measures or both (e.g., Zippert & Ramani, 2017), varying the math instruments used and the kinds of math content assessed (e.g., Susperreguy et al., 2020a, 2020b), examining the relation at different points in development and across time (e.g., Thompson et al., 2017), and exploring the role of socioeconomic disadvantage (e.g., Silinskas et al., 2010).

We hypothesize that a number of factors may have contributed to the inconsistent findings in previous research on the HME and children's math achievement, and we tested these factors as moderators in our analyses. These factors include: 1) HME assessment methods, such as the HME component(s) measured, the HME measurement method used, and how the HME score was calculated, 2) math assessment methods, such as the math domain measured, and whether the math measure was symbolic, standardized, or a composite, 3) study characteristics, such as whether or not the study was longitudinal or concurrent, and 4) sample characteristics, such as age, grade, and socioeconomic status (SES). Table 1 shows the specific coding scheme for all moderators included in the moderator analyses, and the theoretical reasoning behind all moderators is detailed below.

### *HME Assessment Characteristics*

Although theory dictates that the HME should be associated with children' math achievement, the various theories either do not address how the HME should be assessed or have dictated differing methods. As such, the resulting literature has used a variety of methods to assess the HME, from different components highlighted in the measurement (i.e., direct and indirect activities, combined direct and indirect activities, spatial activities, parent socio-emotional factors, and parent math talk), to different measurement methods (i.e., frequency-based scales, rating scales, checklists, and observations), to different ways to calculate the HME score (i.e., latent factor scores, sum or average scores, or a single item). Overall, these between-study differences reveal the large amount of variation in the HME research area, which would benefit from meta-analytic techniques to determine which HME assessment characteristics are differentially associated with children's math achievement.

**HME Component Measured.** Given the sociocultural learning theory, we were interested in exploring the many different ways the HME is defined in the literature, including the more common activities frequency scales (e.g., Benavides-Varela et al., 2016; Hart et al., 2016; LeFevre et al., 2010) to how the math-related social interactions and parent attitudes and expectations toward math within the HME were associated with children's math achievement. Furthermore, Vygotsky (1978) characterized language as the main mechanism for children's learning within these social interactions. Thus, we also considered math talk to be part of the HME and investigated the influence of math talk on children's math achievement. Because the HME is a multi-faceted construct, which has been operationalized in a myriad of ways that vary widely in their associations with math achievement (del Rio et al., 2017; Else-Quest, 2008; LeFevre et al., 2009; Skwarchuk et al., 2014; Zippert & Ramani, 2017), we tested the influence of the HME component measured on the correlation between the HME and children's math achievement in our moderator analysis.

***Direct and Indirect Activities*.** One of the most common components of the HME investigated is the math-related activities that parents and children share in the home. Echoing research conducted on the association between home literacy activities and early literacy skills (Sénéchal & LeFevre, 2002), HME activities are divided into two different types. First, direct or formal activities (henceforth referred to as "direct"), which are explicit instructional activities directly targeting math, like counting or doing math flash cards with your child (e.g., LeFevre et al., 2010). Second, indirect or informal activities ("indirect" going forward), which are everyday activities that incidentally involve math, like playing board games or cooking (e.g., Benavides-Varela et al., 2016; LeFevre et al., 2009; Ramani et al., 2015).

Looking first at direct activities, evidence has shown that explicit math-teaching activities are associated with children's symbolic number knowledge (Skwarchuk et al., 2014) and both concurrent and future numeracy performance four years later (Huntsinger et al., 1998; Huntsinger et al., 2000). However, some studies have also failed to find an association between the frequency of direct math activities and children's math achievement (Blevins-Knabe et al., 2000; Missall et al., 2015). When zooming in on the precise components of explicit math instruction used in direct math activities, LeFevre et al. (2010) found that children improved in their numeration and number sequencing ability when they engaged in direct math activities focused on numbers and basic arithmetic operations more frequently. Some work has also further categorized direct math activities as complex or basic and found that a higher frequency of complex activities, like object arithmetic, quantity comparison and counting by twos, predicted higher math achievement, whereas a higher frequency of basic activities (e.g., counting, reciting numerals, reading and writing numbers) predicted lower math achievement (Skwarchuk, 2009). Finally, there are also several recent studies that categorize direct math activities based on the kind of math skill they target, such as mapping activities, like singing counting songs that ask about quantities, operational activities that involve combining numbers, and patterning activities, which involve using repeated sequences of shapes or numbers (Mutaf-Yildiz et al., 2018; Susperreguy et al., 2018, 2020a, 2020b; Zippert & Rittle-Johnson, 2020). Overall, findings are mixed as to exactly which direct activities are associated with children's math achievement and all the reasons why relations may vary. However, direct math activities are capturing social interactions with a more experienced partner that serve as a prerequisite for children's cognitive development in Vygotsky's sociocultural learning theory applied to a math-specific context. Importantly, these interactions only serve to support math development when they are pushing children toward the upper boundary of their math ability—their zone of proximal development (ZPD)—which may be why only some kinds of direct activities are associated with higher math achievement. In fact, parent reports show that parents of preschoolers engage more frequently in simpler activities, like counting objects than more advanced activities, like comparing numbers and solving arithmetic problems (i.e., an average of once a day versus one to five times per week; Skwarchuk, 2009). This may translate to direct activities only serving those children whose parents are aligning their choice of activities with skills and concepts that are appropriately challenging to support the acquisition of more complex concepts.

There is empirical and theoretical support for the pronounced role of indirect math activities in children's early math development. Research on the home literacy environment, which is in many ways analogous to the HME, has found that indirect literacy activities can teach children to enjoy and appreciate the value of reading (Sénéchal, 2006). Similarly, indirect math activities may help children learn to associate math with fun and shared quality time with parents, providing a positive orientation toward and early interest in math. By virtue of being embedded in everyday context, everyday exposure to a variety of different indirect math-related experiences may be more informative and varied than direct instructional practices for specific math skills. This was supported by a recent meta-analysis of preschool and kindergarten samples, which found that indirect activities had correlations with math achievement that were almost two times larger than direct activities ($r$ = .47 vs. .28; Dunst et al., 2017). Based on the sociocultural learning theory tenet that social learning opportunities within the HME are where children cultivate their math skills when they are pushed toward their upper boundary of math ability, it could be possible that the low-stakes tone of indirect activities allows parents to broach math subject areas that are more advanced than they would feel comfortable addressing more formally but that their children are prepared to learn with help. This would allow them to target their children's ZPD and help advance their children's math understanding without the pressure of deliberate math instruction. Parents who are more comfortable with math may also be more likely to generally embed math into their social interactions with their children more frequently or in ways that make the interaction higher quality, teaching their children the importance of math achievement through casual social interaction.

On the other hand, the fact that indirect math activities do not explicitly target math skills may also mean that they do not effectively support math skill development in the same way as direct math activities. Based on the specificity principle, the development of children's domain-specific skills requires domain-specific inputs in a developmentally appropriate manner (Bornstein, 2002). This means that home learning activities must directly target math to benefit children's math achievement. Because the acts of cooking or playing cards are typically undertaken to have fun and are not often explicitly linked to math learning, they may not satisfy the specificity needed to build up math skills. The specificity principle also posits that within the math domain, children will only develop specific math skills if those specific skills are targeted. For example, if parents want their children to know how to add and subtract, they would need to work on and talk about adding and subtracting directly. This would mean that practicing counting while playing a board game would be targeted enough to cultivate counting skills but would not be specific enough to build up calculation skills. This aligns with the finding that symbolic math skills are related to direct math activities, which are more likely to involve symbolic math instruction, whereas indirect math activities are related to non-symbolic math skills (Skwarchuk et al., 2014) because they are less likely to include a focus on math symbols (i.e., Arabic numerals). This idea is further supported by the finding that direct math activities have a positive, statistically significant correlation of higher magnitude with number identification ($r$ = .42) compared to indirect math activities ($r$ = .18; Vasileyva et al., 2018). Thus, it may also be the case that direct instructional activities that specifically target math are more effective for the development of children's math competencies than indirect activities, leading to a higher magnitude correlation for children's math achievement and direct math activities than for indirect math activities.

In total, findings have been mixed on whether direct, indirect, or both kinds of activities are associated with children's math achievement. Whereas an early study found that only direct math activities were significantly associated with children's math outcomes (Blevins-Knabe & Musun-Miller, 1996), recent meta-analytic work found that indirect math activities are better predictors of children's math achievement than direct activities for samples up to seven years old (Dunst et al., 2017). Other HME studies have shown that both direct and indirect activities are related to children's math achievement (e.g., LeFevre et al., 2009, 2010; Mutaf-Yildiz et al., 2018a; Niklas & Schneider, 2014) and that the distinction between the two is the kind of math they predict, with direct math activities predicting symbolic math skills and indirect math activities predicting non-symbolic math skills (Mutaf-Yildiz et al., 2018a; Skwarchuk et al., 2014). Thus, we tested if assessing the HME component as direct or indirect activities made a difference in the association of the HME with children's math achievement.

***Combined Direct and Indirect Activities*.** Another common method for measuring the HME is to group all HME activities, whether direct or indirect, together. These single component definitions are based on principle component and factor analyses indicating that a single factor is the best way to represent HME activities (e.g., Missall et al., 2015; Purpura et al., 2020). Theoretically, this would mean that it makes is no difference which kinds of math-related activities parents and children are engaging in, but rather it is the fact that they are regularly and frequently sharing quality time together on math in an engaging way that involves the child as an active participant that is supporting a child's math development. This would qualify the interaction as a proximal process, which is the main mechanism of development within the ecological systems theory (Bronfenbrenner, 1979). Parents who can participate in activities that are either directly or indirectly related to math with their children may also regularly dedicate the time and physical and attentional resources to spend quality time with their children to help foster their development. As such, the combined HME activities factor may indirectly be capturing parent availability and willingness to support their children. The findings are mixed, with some studies that use this unitary definition for HME activities finding a positive, statistically significant correlation between general HME activities and children's math achievement as high as $r$ = .42 (Kleemans et al., 2012), whereas other studies have found the relation to be non-significant across a battery of math measures (Missall et al., 2015). Thus, we tested whether HME activities in general had a significant association with children's math skills, and whether or not it was of higher or lower magnitude than more specific characterizations of HME activities or any other HME components.

**Spatial Activities.** Another distinction found in the literature on the HME is the separation of home math activities from home learning activities that are more likely to involve spatial skills, like doing puzzles or playing with blocks (e.g., Hart et al., 2016). Although broader math and spatial skills are related, they also capture distinct skill domains (Zhang et al., 2017). Recent factor-analytic work on the HME found that spatial skills are a separable HME domain from direct and indirect math activities when an overall HME factor is also accounted for (Hart et al., 2016; Purpura et al., 2020), but work outside the HME area has found that spatial skills (i.e., visual–spatial ability) are foundational for the development of later quantitative math skills (Geary & Burlingham-Dubree, 1989). Thus, although spatial activities are empirically separate from other math activities, it is likely that home-based spatial activities could also support children's math skill development in other domains. By virtue of being an important aspect of the ecological system theory's microsystem, the practice of shared spatial activities is likely to be supporting the development of a variety of cognitive abilities, including potentially helping children cultivate their math skills. There may also be parents in certain professions, such as architects or contractors, who are more predisposed to engage in spatial activities, like building with blocks with their children, leading to an exosystem-level influence that ramps up the frequency of spatial activities in the home. In contrast, based on the specificity principle, it is also likely that by virtue of lacking domain-specificity for math outcomes, spatial skills are not specific enough to support the development of children's other mathematical competencies and would instead only support their spatial skill development.

Interestingly, there are four studies on the HME that probed the role of home-based spatial activities in children's math and spatial skills development (Dearing et al., 2012; Hart et al., 2016; Huntsinger et al., 2016; Purpura et al., 2020). One found a non-statistically significant negative relation between spatial activities and spatial skills (when controlling for ethnicity and response bias; Dearing et al., 2012), and another found a negative statistically significant association between a spatial activities factor and parent-reported child math skills (Hart et al., 2016). These negative associations could be artifacts of directionality of causation, with parents being more likely to engage in more spatial activities with children who are struggling with their math or spatial skills. Another study used an achievement measure that was not specific to spatial skills and did not find a statistically significant association between spatial activities in the home and children's math skills (Huntsinger et al., 2016). Finally, a recent study by Purpura and colleagues (2020) found a statistically significant and positive association between a spatial activities factor and children's spatial skills, when an overarching HME factor was included, but the factor did not statistically significantly predict preschool general math performance. Taken together, it is likely that spatial skills are not significantly associated with children's broader math skills. We probed this question using our meta-analytic sample by including a level in our HME component moderator for spatial skills.

**Parent Socio-Emotional Factors.** Much of the recent HME work has expanded the way the HME is defined to also incorporate parental socio-emotional factors about math, such as parent math-related attitudes, beliefs, and expectations for their children's math achievement (e.g., De Keyser et al., 2020; del Rio et al., 2017; Susperreguy et al., 2020a). There are some theoretical reasons to expect parent socio-emotional factors to differ from other HME components. Parent socio-emotional factors have been shown to directly influence parenting practices (Zippert & Ramani, 2017) as well as children's math attitudes and outcomes by reinforcing children's beliefs about their math abilities, which in turn, influence children's math performance (Eccles et al., 1990). Eccles' expectancy-value theory (Jacobs & Eccles, 1992) posits that parents teach their children the value of achievement-related skills through their domain-specific expectations and attitudes. This aligns with Vygotsky's social interaction theory because social interactions with their children are the mechanism by which parents communicate their expectations and attitudes to their children. In turn, these parental beliefs and expectations regarding their children's academic competencies and the importance of specific academic domains (i.e., math) influence their children's academic beliefs and performance in that domain (Bleeker & Jacobs, 2004; Jacobs & Eccles, 1992). This is supported by both cross-sectional and longitudinal research showing a statistically significant role for parent math-related self-efficacy (i.e., math confidence) and math anxiety (Jameson, 2014), as well as parents' views of their children's math competencies (i.e., math expectations) in children's math outcomes and beliefs (Bleeker & Jacobs, 2004; Simpkins et al., 2012).

Meta-analytic work on general parent involvement and academic achievement shows that compared to many other forms of parent involvement, including direct activities like parent homework help, parental aspirations and/or expectations had the strongest association with children's general academic achievement (Fan & Chen, 2001; Jeynes, 2007). This is supported by work showing a higher magnitude correlation for broad math skills with parent math expectations than with parent math activities ($r$ = .63 versus $r$ = .41; Segers et al., 2015). However, studies with primary school children have also shown mixed results, both supporting the statistically significant role of parent math expectations in children's math achievement (del Rio et al., 2017) and failing to support it (Susperreguy et al., 2020a). In addition, a meta-analysis on the HME-achievement link found that, when examining parent education, family SES, parent attitudes toward math, parent expectations for math, and home numeracy experiences (i.e., activities), home numeracy experiences were the best predictors of preschool and kindergarten children's math performance (Dunst et al., 2017). Our moderator analyses enabled us to determine whether activities or parent-socio-emotional factors had a more pronounced role in children's math achievement compared to other HME components, like direct math activities.

**Parent Math Talk.** One more line of inquiry often used in the study of home and parental influences on children's math achievement, which has not traditionally been combined with other measures of the HME, is "parent math talk." Parent math talk refers to parent utterances of number words (e.g., one, two; also called "parent number talk") and words related to magnitude comparisons (e.g., more, less; Gunderson & Levine, 2011; Levine et al., 2010) and starts during their children's infancy and early toddlerhood (Durkin et al., 1986). Math talk measures quantify math language use by counting up the number of times parents make math-related utterances based on observations of parents and children either in the home or a more controlled laboratory setting (e.g., Levine et al., 2010). Additionally, math talk measures sometimes account for supportive forms of communication happening in conjunction with math language use, like using objects to demonstrate math-related utterances (e.g., Gunderson & Levine, 2011). Similar to indirect math activities, math talk captures math-related interactions that are embedded in day-to-day activities, like shopping, cooking, and family meals (Walkerdine, 1988). For instance, parents may compare how much items cost or weigh at the grocery store, talk about fractions when measuring ingredients to cook, discuss how long dinner lasts or how many people, plates, or food items are at the table. The difference, however, lies in the fact that math talk is directly capturing math language use, rather than just the existence of a math-related interaction. Given that Vygotsky (1978) identified language as the main mechanism for children's learning in social interactions, and that both general language (Purpura & Ganley, 2014) and math-specific language (Purpura et al., 2017) are related to children's math outcomes, it is important to include parent math talk in the HME.

The quantity of parent math talk has been shown to be associated with children's numerical knowledge (Levine et al., 2010). More specifically, parent math talk predicts children's later number knowledge when math talk is advanced rather than basic and involves talking about numbers 10 or higher (Elliott et al., 2017), counting, labeling the cardinal value of visible objects, talking about large sets of objects (Gunderson & Levine, 2011), and talking about ordinal relations (Ramani et al., 2015). The reason for this association is most likely because when children are able to see the numbers they are counting and hear large numbers it helps them link number words and the cardinal value of sets. Thus, parent math talk may be essential for creating mental representations that link math language with mental conceptualizations of math concepts.

**HME Measurement Method.** There are four main measurement methods used for the HME: frequency-based scales, rating scales, checklists, and observations. These different ways of quantifying the same phenomenon may cause differences in HME scores' adequacy in meeting statistical assumptions, variability, and potential for bias. Frequency-based scales ask questions about how often parents engage in indirect or direct math-related activities in the home, such as parent-child board game play or doing math workbooks based on a 4- to 7-point Likert scale. The frequency can range anywhere from several times a day, to daily, monthly, or never, which represent inconsistent time intervals and do not provide an indication of quality. Frequency-based measures tap into the prerequisites of proximal processes in Bronfenbrenner's ecological systems theory: the need for shared learning experiences to be frequent, regular, and involve the child as an active participant. Frequency-based scales operate under the assumption that more frequent math-related interactions are always better than less frequent ones. However, because ordinal scales operate under the statistical assumption that the difference between two scores represents a difference in rank order of quality, it may not be proper to rank more frequent interactions as “better” than less frequent interactions (i.e., giving daily versus weekly interactions more points). Additionally, there may be some cases where less frequent interactions, like a two-hour weekly math tutoring session, may be higher quality than a ten-minute flash card review session that happens daily. Accordingly, a score increase on these scales does not directly translate to an increase in rank order, and this could affect the correlation of the HME to children’s math achievement.

Sometimes HME activities are measured by checklists (rather than frequency scales), on which, for example, parents indicate all the number game titles with which they are familiar (e.g., Bhanot & Jovanovic, 2005; LeFevre et al., 2017; Skwarchuk et al., 2014). This checklist of number game exposure is then used as a proxy for the overall level of indirect math learning in the home by attempting to take a holistic snapshot of how much informal math exposure present in the home without fully capturing how routine this exposure may be. Overall, checklists are based on the idea that games with math content have been shown to support the development of math competencies in children (e.g. Ramani & Siegler, 2008). By contrast, frequency-based scales are quantifying the emphasis and importance placed on math learning and what math *means* in a given environment. Frequency scales were created under the assumption that the activities engaged in most frequently within the home are the embedded with the most meaning. Checklist scales often include foils that attempt to capture social desirability in parents’ responses (e.g., Susperreguy et al., 2020b), with total scores calculated based on the total number of correct number games titles indicated minus a penalty for all fake number game titles marked.

Alternatively, sometimes rating scales are used to measure the HME. These typically ask parents to rate socio-emotional factors based on their agreement with or the degree of importance they give to a given statement about math. For example, for parent attitudes/beliefs toward math, parents indicate their level of agreement with statements about how much they enjoy or avoid math (Skwarchuk et al., 2014). For parent math expectations, parents rate the importance they place on their children reaching certain math-related benchmarks by a given age or grade (e.g., “To what extent do you expect your child to have mastered the following skills at the end of kindergarten?”) on skills like counting to 100 (Kleemans et al., 2012). As such, a score increase on these scales translates to an increase in rank order, even if each unit increase does not necessarily represent the same quantity. Rating scales capture the important mesosystem influence that links the home and school by indicating to children how much work they should put into and how much emphasis they should place on their math skill development. In terms of parent math expectations, rating scales are also capturing a more holistic picture of the HME because they are embedded with the chronosystem influence of time. Parents indicate the benchmarks they believe their children should be meeting by a certain age, and those expectations (and ratings) fluctuate based on the child’s point in development. Furthermore, rating scales tend to allow for a larger range of values (i.e., 0 to 10 or even 0 to 100 in Hart et al., 2016), potentially resulting in higher variability than the previous frequency scales or checklists. This increased variability and consistent interval measurement may result in higher magnitude correlations between the HME and children’s math achievement for HME instruments that use rating scales.

The measurement techniques discussed so far are all parent- or child-report questionnaires, but the HME is also sometimes assessed using observation-based measures. Observation-based measures are frequently used to capture parent math talk (e.g., Ramani et al., 2015; Susperreguy & Davis-Kean, 2016) and are sometimes used to measure direct and indirect HME activities, like math homework help and cooking (e.g., Lindberg et al., 2008; Vandermaas-Peeler et al., 2012). There are a number of factors that may drive differences in self-report versus observation-based measures. For example, social desirability may cause parents to respond to survey measures in ways that make them appear more favorable, like inflating the frequency they report for shared math activities. It is also possible that parents have trouble accurately recalling and reporting their math-related interactions with their children, which may lead them to guess and over- or under-estimate their reports. Thus, observation-based measures may capture a more accurate picture of the HME. There is also the potential for observer bias to cause parents to act in ways that appear more supportive and helpful than they would typically act in day-to-day life. However, the accuracy of an observer report is still likely to be superior to the accuracy of a recall-based parent-report measure.

Based on the theoretical and methodological reasons outlined above, we tested for HME measurement method as a moderator of the relation between the HME and children's math achievement. Additionally, based on a recent systematic review calling for more research directly comparing questionnaires to observational methods (Mutaf-Yildiz et al., 2020), we collapsed the HME measurement method moderator into these two basic levels. This allowed us to test whether the differences in HME-math achievement effect size estimates were partially attributable to these two basic assessment types, rather than more nuanced measurement differences.

**HME Score Calculation.** Researchers also differ in the ways they calculate HME scores. Many studies use exploratory factor analyses to create one or more latent factors representing the HME, but other studies use sum or average scores from the HME assessment items, or simply analyze single HME questions. Based on the reduction in measurement error provided by latent factors in comparison to measured variables (Gayán & Olson, 2003), we expect to find higher magnitude correlations when the HME is measured as a latent factor, rather than through the use of sum or average scores or by using single HME questions. In order to account for the impact of these different HME calculation methods on the association between the HME and children's math achievement, we tested the HME score calculation method in our moderator analyses.

### *Math Assessment Characteristics*

Given our consideration of the potential impact of differences in HME measurement, it was also important to consider the impact of differences in math measurement on the HME-math achievement link. We tested moderators related to math domain, math composite, symbolic versus non-symbolic and timed versus untimed nature of the math measure, and if the measure was from a standardized assessment.

**Math Domain.** The math achievement literature consistently shows that math ability is made up of many component skills that are related yet distinct (e.g., Purpura et al., 2017). Thus, the vast variability in the associations found between the HME and children's math achievement may be partially due to the myriad ways in which math is measured, like the math domain assessed. Indeed, the majority of the research on the HME and children's math achievement has investigated a wide array of math skills and spatial skills (e.g., Purpura et al., 2020; Thompson et al., 2017).

The majority of studies show that the direction, strength, and significance of the association between the same HME component and children's math achievement differs between math domains (e.g., Dearing et al., 2012; Huntsinger et al., 2016; Kleemans et al., 2013; Missall et al., 2013; Mutaf-Yildiz et al., 2018a, 2018b). Many HME studies find different HME-achievement relations when they assess a number of different individual math domains separately (e.g., Susperreguy et al., 2020b), ranging from more basic math skills like counting and number naming (Zippert & Ramani, 2017) to more advanced math skills like arithmetic operations (Kleemans et al., 2013). For example, Zippert & Ramani (2017) found that basic math skills, like numbering, were not significantly associated with the HME, but more complex math skills, like arithmetic operations, were. However, in other instances, statistically significant, positive associations have also been shown for the HME with basic, counting-related measures (e.g., Cheung et al., 2018; Manolitsis et al., 2013).

Because most HME research focuses on young children prior to entering formal schooling, the bulk of studies on the HME-math achievement link tend to be focused on informal numeracy skills, which can be grouped under the overarching categories of numbering, (numerical) relations, and arithmetic operations (National Research Council, 2009; Purpura & Lonigan, 2013). Numbering refers to understanding of the rules and processes associated with counting, including verbal counting, counting errors, one-to-one correspondence, cardinality, subitizing, and estimation. Relations refers to understanding of the ways two symbolic numbers may be associated, including quantity comparison, number comparison, number naming, ordinality, and number line sequencing. Arithmetic operations refers to knowledge of the ways sets and subsets of numbers can be created and decomposed, including addition, subtraction, and other forms of combining numbers.

Notably, our categorization of math domains was similar to the math outcomes tested in the recent meta-analysis on the HME-achievement link by Dunst and colleagues (2017), which assigned math assessments to three categories: simple, basic and complex. Their classification for simple math outcomes maps onto the numbering domain tested here, whereas both basic and complex math outcomes map onto arithmetic operations (with complex including more advanced operations like multiplication), so our relations domain represents a novel way of accounting for differences in the math domain assessed. Based on their findings that all three domains had significantly different effect sizes and that effect sizes that include the basic math measures (numbering for the present study) were the largest ($r$ = 0.57) and those that included the simple math measures were the smallest ($r$ = 0.20), we expect to find the same pattern in our results with effect sizes that assess the numbering math domain showing the highest magnitude correlations or at least higher correlations than the arithmetic operations domain. However, because their sample was limited to only preschool/kindergarten children, and our sample includes a wider range of ages and grades, we may find a different pattern of relations here as well. We included the math domain moderator in order to reveal the influence of the chronosystem on children's math outcomes, which captures the vital role of timing on children's skill development. Based on the generally young samples used in HME research, the more basic math domains may be more closely associated with the HME than more advanced math domains, due to the alignment of children's abilities and input in terms of timing for their point in development. Furthermore, parents who hold certain occupations, like engineers or other STEM careers, may be more likely to engage in math interactions that include more advanced math domains and have higher math expectations and math confidence than parents who do not have a math- or science-based career, and the math domain moderator will allow us to capture this exosystem-level influence. Finally, research on math talk has revealed that low- and middle-income parents tend to use more basic math talk about sizes of objects, numeral identification, and rote counting than math talk about advanced concepts like arithmetic, so the input children are receiving may be more specifically targeting certain math domains (and points in development) than others (Levine et al., 2010).

Our math domain moderator analysis helped us parse these differences among math domains.

Additionally, we also tested for the possibility of a Math Domain x HME Component interaction in case these differences are not solely due to math domain but to the match-up between a certain HME component and a certain math domain based on evidence that direct and indirect home numeracy activities are differentially related to specific mathematical tasks (Mutaf-Yildiz et al., 2018a; Vasilyeva et al., 2018; also reviewed in the HME component section). Based on the meta-analytic finding that the influence of math domain on the HME-achievement link differs as a function of age (Dunst et al., 2017) and the chronosystem influence of timing on child outcomes, we also conducted a Math Domain x Age interaction analysis to assess the same interaction in our larger, older sample.

**Composite.** Many studies also use comprehensive assessments that capture a full spectrum of math skills, including both basic and advanced math skills, at once (e.g., Napoli & Purpura, 2018; Segers et al., 2015). A number of these studies have shown high magnitude correlations between the HME and children's math achievement, ranging from $r = .29$ to as high as $r = .63$, indicating that comprehensive math assessments may be best to capture the role of the HME in children's math achievement compared to single skill assessments (e.g., Napoli & Purpura, 2018; Segers et al., 2015; Thompson et al., 2017). Thus, it is also possible that comprehensive, multi-domain math assessments are capturing the role of the HME more fully than assessments that measure a single domain, resulting in higher magnitude correlations for composite as opposed to single assessments. By including whether or not a math assessment is a composite in our moderator analyses we answered the call made in a recent systematic review (Mutaf-Yildiz et al., 2020) to examine whether composite math scores or single domain math scores may be better for use in future HME research.

**Symbolic.** A few studies have also shown differing links between the HME and children's math achievement when the associations include math assessments that represent numerical magnitudes symbolically versus non-symbolically. There are fundamental differences between symbolic versus non-symbolic number representations. Non-symbolic numerical magnitudes (i.e., dot arrays or groups of objects) are easily discernable, measurable in early infancy, and presumed to be recognized by all species (Cantlon, 2012). On the other hand, symbolic representations (i.e., Arabic digits) provide abstract representations of numerical magnitude and are uniquely human. In HME research, it is common for non-symbolic math ability to measured using objects for arithmetic or magnitude comparison (e.g., Missal et al., 2015; Skwarchuk et al., 2014), whereas symbolic skills are measured using more traditional math instruments like paper- or computer-based arithmetic assessments or flashcards with Arabic numerals on them.

Some studies have shown that direct math activities were associated with preschool children's symbolic number skills but not their non-symbolic skills, and the opposite was true for indirect activities (Skwarchuk et al., 2014; Susperreguy et al., 2018). In a different preschool sample, Mutaf-Yildiz et al. (2018a) also found a lack of association between direct activities and non-symbolic number processing but found that both direct and indirect activities were associated with symbolic number skills. When the HME-math achievement association was probed via path analysis, Wei et al. (2020) found that symbolic and non-symbolic number estimation ability each uniquely predicted children's initial math achievement, with identical beta values (.06), indicating that differences in the HME-achievement link may not be driven by whether the math assessment is symbolic or non-symbolic. Based on these findings, we expected a statistically significant association between direct activities and symbolic number skills and indirect activities and non-symbolic number skills but were uncertain about the other patterns that would emerge showing differing correlations with the HME for symbolic versus non-symbolic math skills. Based on the social learning theory and the principle of specificity, the social interactions that support math learning in the home microsystem need to expose children to both symbolic and non-symbolic number knowledge to prepare them to encounter both kinds of math skills in school, thus providing a mesosystem-level influence that links the home learning environment with the school learning environment. Based on evidence that symbolic number knowledge is strongly related to general math achievement for children younger than six (Fazio et al, 2014), we may find that the HME-math achievement link is especially strong for symbolic number skills. Our symbolic moderator analyses tested whether there were differences in the association between the HME and children's math achievement based on whether symbolic or non-symbolic math assessments were used. In addition, based on HME work showing patterns of differing relations for different HME components and a recent systematic review calling for more research to clarify which specific type of HME activity is linked with which specific type of math skill (Mutaf-Yildiz et al., 2020), we also tested the interaction between the HME component measured and symbolic or non-symbolic math assessments.

**Standardized.** The differences found in the HME-math achievement link could also be due to differences in math assessment related to whether the math measure was standardized or unstandardized (i.e., researcher-created). Meta-analyses on intervention research across many content areas have shown that intervention effects have larger effect sizes for researcher-created assessments, compared to standardized assessments (Wolf et al., 2020), with researcher-developed assessments showing effect sizes that are 0.20–0.29 standard deviations greater than standardized measures due to researcher-created assessments usually being tailored to the skills targeted in the intervention (Cheung & Slavin, 2016; de Boer et al., 2014; Li & Ma, 2010). Because the HME is an informal form of intervention, in which parents play a role in helping their children develop their math skills, we expected that the magnitude of the association between the HME and children's math achievement may be higher for unstandardized (i.e., researcher-created) assessments than standardized ones. By virtue of being norm-referenced, standardized tests also tend to be higher quality and more reliable than less objective (i.e., researcher-created) measures, which have not been administered to a normative sample and may be more easily influenced by assessor biases. Meta-analytic work on parent involvement for both elementary and secondary urban students has shown a greater relationship between parental involvement and grades and other unstandardized measures compared to standardized tests (Jeynes, 2005, 2007). This could be attributable to the fact that parental involvement is usually focused on classroom-based assignments and assessments rather than preparing for standardized tests and would likely translate to a higher magnitude correlation between the HME and children's unstandardized math test performance than their standardized math test scores. Furthermore, it is likely that researcher-created assessments are tailored to the specific skills being targeted in the HME, and based on the specificity principle, this honed-in focus will be better suited for capturing the development of specific skills. As a result, it was important for us to examine the potential moderating effect of standardized versus unstandardized assessments on the HME-math achievement link.

### ***Study Characteristics***

**Longitudinal or Concurrent.** Whether or not a study captured longitudinal (at different time points) or concurrent (at the same time point) relations between the HME and children's math achievement may also moderate the correlation found. For example, the benefits conferred by the HME for children's math achievement may weaken over time (Manolitsis et al., 2013). On the other hand, the effects of the HME may take time to be reflected in children's math performance, resulting in stronger effect sizes for longitudinal associations compared to studies that measure the HME and children's math achievement concurrently. According to Bronfenbrenner's chronosystem, the influence of time and timing of specific learning experiences impacts how and to what degree children's math skills develop (Bronfenbrenner, 1979). Based on a recent meta-analysis on preschool and kindergarten samples, which found substantially larger effect sizes for longitudinal assessments that were measured at least 18 months apart than for concurrent assessments or assessments measured up to three to six months apart (Dunst et al., 2017), it is possible that we will find larger effect sizes for longitudinal studies than for studies that administered concurrent assessments. This is also supported by recent work showing that the HME predicted later but not concurrent math skills (Niklas & Schneider, 2014; Susperreguy et al., 2020a), and that children's math achievement growth but not their initial math achievement status was associated with the HME (Wei et al., 2020). Given this possibility and supporting evidence, we accounted for whether or not a study included an effect size that captured longitudinal or concurrent relations in our moderator analysis to capture the impact of specific timing on the association between the HME and children's math achievement. Importantly, the influence of timing was also found to differ as a function of age (Dunst et al., 2017), so we tested that possibility here with an Age x Longitudinal interaction analysis.

### *Sample Characteristics*

**Age and Grade.** In an effort to capture the effects of the HME prior to formal schooling, the majority of HME research has been conducted on preschool and kindergarten samples. This focus on parental involvement in the early years aligns with meta-analytical results showing that general parental involvement has higher magnitude associations with children's achievement in earlier rather than later grades (Jeynes, 2007; Patall et al., 2008). However, this expected developmental pattern (i.e., the correlation between children's achievement and parental involvement declines over time) is not clear-cut in the HME research area. In fact, both positive and negative and significant and non-significant relations between the HME and children's math achievement have been found in both younger and older samples (Ciping et al., 2015; LeFevre et al., 2009; Pezdek et al., 2002). Negative relations between the HME and children's math achievement may be due to parents increasing math-related interactions with their children when they notice that their child is having difficulties with math (Ciping et al., 2015). It may also be due to the fact that older children entering adolescence are likely to be building their autonomy, leading older children to refuse parental assistance and potentially start declining in mathematical competence. Additionally, older children's parents may be unable to provide adequate math support for the challenging content their older children are learning (Gutman & Midgley, 2000), resulting in less math support overall and potentially non-significant or weaker relations between the HME and children's math achievement because parents are not equipped to help with their children's math in a way that supports their development.

In contrast, younger children's skills are still growing and developing, and parents can more easily master the concepts younger children are learning (Patall et al., 2008), resulting in more math support overall and potentially stronger correlations between the HME and children's math achievement. This is supported by meta-analytic work on general parental involvement and children's overall academic achievement, which shows a greater effect size for elementary school children compared to secondary school children (Jeynes, 2005, 2007). Similarly, the meta-analysis on preschool and kindergarten children also found that the correlation between the HME and children's math achievement was nearly three times higher for preschool children than those in kindergarten (r = .57 vs. .17; Dunst et al., 2017). Applying this evidence to math-specific learning and achievement in a sample with a wider variety of grades, we expect children in lower grades to show higher magnitude correlations between the HME and math achievement for children in lower grades than children in higher grades. To build on the evidence proffered by the findings of Dunst et al. (2017), our categorization of sample grades will include two subcategories: one for preschool and kindergarten children and one for children in primary and secondary grades.

Recent work by Thompson and colleagues (2017) showing that advanced HME activities were statistically significantly associated with four-year-olds', but not three-year-olds', math skills has also demonstrated that nuanced differences between children in the same grade but different ages may also exist. By conducting both age and grade moderator analyses, enabled us to detect more nuanced developmental differences in the relations between the HME and children's math achievement. Furthermore, if both the age and grade moderator variables showed the same pattern, the results of each would provide supportive evidence for the same moderating effect across development.

The specificity principle also highlights the need for domain-specific inputs to be developmentally appropriate (Bornstein, 2002), which aligns with Vygotsky's concept of a zone of proximal development (ZPD). This means that even a parent-child exchange formally targeting math skills is only valuable for a child if the skills targeted are advanced enough to push the child beyond his or her current level of functioning (i.e., the child's ZPD). One seminal study has addressed this distinction between advanced and basic formal math activities and found support for the specificity principle, showing that only advanced formal HME activities (which are developmentally more advanced than informal HME activities) were statistically significantly associated with preschool children's math achievement (Skwarchuk et al., 2014). However, the distinction between advanced and basic math activities is not universal in the HME-math achievement literature.

A potential way to test for whether the specificity principle requirement for developmental appropriateness holds is to examine whether there is an HME component x Age and/or HME Component x Grade interaction. This would reveal whether children's developmental time point, represented by age and/or grade, affects the association between the HME and children's math achievement based on the specific aspect of the HME being measured. For example, it may be the case that math activities have a higher magnitude association for younger children, like the preschool sample in Skwarchuk et al. (2014). It may also be the case that parent math expectations have a higher magnitude association with children's math achievement for older children (in higher grades) because the role of math expectations is only revealed over time after math skills can build up.

**Socioeconomic Status (SES).** Years of research have supported the existence of SES-related math achievement gaps that appear before the onset of formal schooling (Aunola et al., 2004; Starkey & Klein, 2008) and widen during early childhood (Starkey & Klein, 2008). One potential explanation for this gap is that socio-economically disadvantaged families have less access to resources needed to provide math-related support within the home, including less time to provide stimulating interactions to teach school readiness skills, like numbers and (math) language (e.g., Davis-Kean, 2005; Hart & Risley, 1995). This may be due to the exosystem-level influence of low-SES parents holding multiple jobs to make ends meet amid socioeconomic strain, leaving them with less discretionary time to spend helping their children with math and communicating their math attitudes and expectations to their children (Lareau, 1987). Furthermore, evidence from observing child-parent dyads during play shows that, in comparison to low SES parents, high SES parents provide more math-related exchanges during play time (Vandermaas-Peeler et al., 2009). In addition to an increased likelihood of spending time with their children, high SES parents are also more likely to be more prepared to teach literacy and numeracy skills (Blevins-Knabe & Musun-Miller, 1996). In fact, evidence shows that low SES parents tend to underestimate their children’s potential for math ability, with middle SES parents setting higher goals for shared at-home math activities (Saxe et al., 1987) and having more accurate math expectations for their children’s point in development (Starkey & Klein, 2000), making middle-SES parents more likely to engage in activities within children’s ZPD (DeFlorio & Beliakoff, 2015). These differences in family provision of math learning and understanding of children’s math learning potential and accurate ZPD estimation could potentially manifest as SES-driven magnitude differences in the HME-math achievement link, with low SES households showing a lower magnitude correlation than high SES households. Thus, we investigated the potential moderating effect of SES on the correlation between the HME and children’s math achievement with two different approaches. One approach was a categorical SES indicator capturing family SES based on parental income, and the second approach was a continuous SES indicator based on the highest level of education attained by the parent(s).

Additionally, evidence also shows that the moderating effect of HME component on the HME-achievement link may be moderated by the SES of the sample. For example, one study found that low SES children showed a statistically significant association between board game playing frequency (i.e., indirect math activities) and math achievement but high SES children did not (Ramani & Siegler, 2008). Further evidence has shown that low SES children who have lower levels of math achievement also tend to be engaged in less complex number activities at home (Saxe et al., 1987). The pattern of these associations indicates that the correlation between certain HME components and children's math achievement may vary based on their SES. Accordingly, we also tested the interaction between SES and HME component to examine whether differences in HME-achievement associations for low versus high SES children were driven by the HME component measured.

## Study Aims

The purpose of this preregistered meta-analysis was to investigate two main research aims. Our first study aim was to combine all previous studies conducted on the association between the HME and children's math achievement to calculate the average weighted correlation between the two constructs. Our second study aim was to empirically test a range of potential moderator variables that may influence the strength of the relation between the HME and children's math achievement in order to explain the heterogeneity across studies in the HME-achievement association.

# Method

In the current meta-analysis, we synthesized all available empirical evidence on the relation between the HME and children's math achievement. All aspects of the study followed a preregistration plan (Prospero ID # CRD42018099626), unless explicitly noted. No IRB approval was needed for this study.

## Literature Search

The literature search was conducted in the fall of 2018 with no publication date constraints on eligible articles. To begin, we searched EBSCO Discovery Science, a search tool that draws from all university-accessible databases. We used the search phrase (“home numeracy environment” AND parent* AND home), and annotated which databases came up in the results. Two expanders were added to this search, namely “also search within the full text of the articles” and “apply equivalent subjects.” The databases included: Education Source, Academic Search Complete, Education Full Text (H.W. Wilson), Social Sciences Citation Index, Academic OneFile, Education Resources Information Center (ERIC; including theses and dissertations), Child Development & Adolescent Studies, MEDLINE (PubMed), MEDLINE (ProQuest), PsycARTICLES, PsycINFO (including PsycINFO Theses and Dissertations), and Social Sciences Full Text. Each database was searched independently with the following comprehensive search terms: ("home math environment" OR "math talk" OR home OR "home environment" OR "home learning" OR "home experience" OR "home numeracy" OR "informal learning environment" OR "home practices" OR "home activities") AND ("parent child interactions" OR "parent school relationship" OR "parent characteristics" OR "parent expectations" OR "parents as teachers" OR "parent student relationship" OR "parent child relations" OR "parent attitudes" OR "parent beliefs") AND ("number activities" OR "number skills" OR numeracy OR "early numeracy" OR math* OR "math skills" OR "math ability" OR "mathematical reasoning").

Next, additional databases were chosen by reviewing the databases listed on Florida State University’s library research guides for related topic areas in Psychology, Mathematics, Education, Early Childhood Education, and Family and Child Sciences. Based on these guides, three additional databases were included: Educators Reference Complete, MathSciNet, and Web of Science. In order to be as comprehensive as possible and capture literature that may not be found in topic-specific databases, Google Scholar was also searched.

Once the database searches were all conducted, the results were saved in RefWorks to be reviewed for inclusion in the present meta-analysis. First, articles were excluded based on duplicates (which RefWorks automatically detects). Second, all non-duplicates were reviewed by the first and second authors to determine whether studies met inclusionary criteria based on titles and abstracts, and in cases where it was necessary, a review of methods, tables, and/or full manuscripts was also conducted to determine whether inclusionary criteria were met.

Once the final sample of articles was determined, reference lists were then reviewed in order to determine whether there were any articles cited by an included article that did not appear in the previous searches. The references were also reviewed to identify prominent authors in the area that had multiple publications or a seminal publication in the HME research area (i.e., a paper that was highly cited, had methods that were highly replicated, or developed a frequently used HME measure). The Google Scholar profiles of prominent authors in the area were also reviewed to make sure all their relevant work was captured.

Then, in an effort to procure unpublished work on the association being investigated and comply with reviewer requests, we also conducted a grey literature search that was not part of the preregistration. Specifically, we posted messages on Twitter, the Cognitive Development Society (CDS) listserv, and the Math Cognition and Learning Society (MCLS) listserv requesting that researchers with unpublished data on the association of the HME and children's math achievement in the form of theses, dissertations, conference submissions, presentations, or posters, and manuscripts in preparation or under review provide their data to be included in the present meta-analysis. In addition, past conference programs from CDS and MCLS were manually searched for conference talks, posters, or spoken papers with titles and abstracts related to the association between the HME and children's math achievement. This led to a total of twelve first authors being emailed requests to share their work in mid-January 2020. Only two authors replied and provided the correlation, sample size, and study details necessary to code for moderators in order to add their unpublished work to the final meta-analytic sample. Finally, also in mid-January 2020, all members of a recently established network of prominent HME researchers were contacted directly via email to request unpublished work on the HME-achievement link. They included Tom Gallagher-Mitchell, Victoria Simms, David Purpura, Erin Maloney, Thomas Hunt, Camilla Gilmore, Gerardo Ramirez, Jo-Ann LeFevre, Susan Levine, Rose Vukovic, Pamela Davis-Kean, Abbie Cahoon, Bert de Smedt, Bert Reynvoet, Frank Niklas, Alexa Ellis, and María Inés Susperreguy. Combined, all grey literature search efforts yielded a total of 22 new study samples examining the association between the HME and children's math achievement to be added to our final meta-analytic sample.

**Inclusionary and Exclusionary Criteria**

To be included in the present meta-analysis, a primary study had to meet the following criteria:

1. A study must have had an operationally-defined HME measure. The HME must have measured practices, parent talk, attitudes, expectations, and/or beliefs that are math-specific separately from other achievement domains (e.g., literacy, science) or the study was excluded. For example, studies that only used a general home learning environment measure (e.g., Casey et al., 2014; Hindman et al., 2010; Foster et al., 2016; Galindo & Sheldon, 2012), without separating math-specific aspects, were not included. Studies that measured home math talk (e.g., Ramani et al., 2015) and parent attitudes, beliefs, and/or expectations about math (e.g., del Rio et al., 2017; Segers et al., 2015), or parent math

talk (e.g., Gunderson et al., 2011) were included in our conceptualization of the HME, but only if they were math-specific.

2. A study must have included at least one math-specific achievement measure that did not include other achievement domains (e.g., language skills; Keith & Lichtman, 1994) in order to isolate the effect of the HME on math achievement only. The math achievement measure could have involved any assessment method, including parent-report of children's math skills (Hart et al., 2016), and standardized (e.g., Blevins-Knabe & Musun-Miller, 1996; Cheung et al., 2017) and unstandardized (e.g., LeFevre et al., 2009; Skwarchuk et al., 2014) math tests. Studies that examined the HME but did not have a math achievement measure were excluded (e.g., Anderson, 1997; Missall et al., 2017).
3. If a study reported more than one math achievement outcome, the same math achievement outcome at multiple time points, and/or had more than one component of the HME included, the multiple combinations of HME measure and math achievement measure were included as separate effect sizes.
4. If a study did not report the zero-order correlation between the HME and a math achievement outcome, reported a partial correlation that controlled for other variables, or did not report sufficient statistics to allow us to derive a zero-order correlation between the two, then the primary corresponding author of the study was contacted via E-mail in an attempt to procure the missing information. If the author did not respond within two weeks or chose not to provide the information, the study was excluded. A consequence of this criterion was that only quantitative studies examining the HME-math achievement relation were included in the present analysis and all qualitative studies were excluded.

**Coding Procedures and Reliability**

For the present meta-analysis, we implemented a systematic process for identifying and coding the study results (i.e., Pearson correlation coefficient and corresponding sample size) and study descriptors (i.e., moderators) from the primary studies (outlined in Table 1). The coding was done in four phases by three authors of this article.

In the initial coding phase, the first author chose the inclusionary and exclusionary criteria and conducted all article searches for a Meta-Analysis class she took in fall 2018 under the supervision of a professor with expertise in conducting meta-analyses. Then, the first author recruited the second author to assist in the article selection and coding process due to her experience in HME research. The article selection and elimination was done by the first and second authors. Training for coding was conducted remotely via FaceTime to review, define, and elaborate on all inclusionary and exclusionary criteria and all moderator coding schemes. Example papers were provided and each step was reviewed in depth, until all questions were answered and the second author was prepared to begin the article selection and coding process. In order to collaborate remotely, the first author created a shared Google Drive with the second author, which included all articles to be coded or eliminated and a folder for each designation, a document for keeping track of articles that were rejected and the reason(s) why, and an electronic codebook that included detailed moderator coding schemes. The first and second author individually coded five primary studies, which was followed by a comparison of their coding consistency and discussion of any issues that needed clarification or verification before moving onto the next phase of individual coding.

In the second phase, the first and second authors reviewed and made all article rejections. There were no articles for which the choice to include or exclude was unclear, as most exclusions were made on the basis of studies that either did not report correlations (for which data could not be procured through author query) or included a home learning environment or child math achievement measure that was not math-specific.

In the third coding phase, the first and second authors extracted all effect size and moderator data from each article included in the final sample. All questions that arose during the data extraction phase were addressed via email correspondence between the first and second author until a unanimous decision was made on every uncertainty that arose.

The fourth and final coding phase involved re-coding every article in the study sample so that every study was coded twice among the first three authors. This was done in response to reviewer request to double-check all coding. Then, the quality of the coding was evaluated by the average inter-rater reliability testing using Cohen's kappa (Cohen, 1960) on a random selection of approximately 50% of the articles from the final sample ($n$ = 30). A Cohen's kappa of .89 was achieved, indicating a high inter-rater reliability and that the data were ready for analysis.

***Coding Procedures***

The coding scheme included: Author(s), year of publication, HME component, HME measurement method, HME score calculation, math domain, whether or not the math assessment was a standardized, symbolic, or a composite, whether or not the study was longitudinal, average age of the sample, average grade of the sample, and average SES of the sample (both as an SES factor and continuously as parent education). See Table 1 for the detailed coding scheme. Two columns were also created to code for correlations that came from the same article and study sample (i.e., study ID) and to code for differences in publication type. These final data were then imported into `R Version 3.5.3` (R Core Team, 2020), and all Pearson's correlation coefficients ($r$) and the corresponding sample sizes ($n$) were used to calculate the corresponding Fisher's $Z$ and variance for each effect size using the escalc() function from the `metafor` package (Viechtbauer, 2010). Then, Fisher's *z*-transformed effect size was utilized for all subsequent analyses in `R` and converted back to *r* for all reporting. All changes made to moderator coding, which did not precisely follow the current report's preregistration, and the reasons behind those changes are detailed in supplemental materials.

***Coding details***

The detailed coding schemes for each individual moderator analysis are included in the supplemental materials.

**Effect Size Computation and Combining Effect Sizes**

All the following analyses were conducted using the `metafor` package (Viechtbauer, 2010) in `R version 3.5.3` (R Core Team, 2020).

***Overall Average Effect Size***

We examined the average association between the HME and children's math achievement using the zero-order correlation coefficient, or *r* effect size. This *r* effect size was chosen because the empirical work examining the relation between the HME and children's math achievement uses primarily correlational designs that report Pearson correlations. Some studies in the final sample used experimental designs that employed HME interventions (e.g., Peters, 1998) and compared the math performance of those who participated in the intervention to a control group. In these cases, we reported only concurrent effect sizes that were calculated before the intervention was implemented. Once the *r* correlation coefficient(s) between the HME and children's math achievement was coded for each study, the effect sizes were converted using Fisher's *Z* transformation (Lipsey & Wilson, 2001) to approximate a normal distribution of population effect sizes (Cohn & Becker, 2003). We then used the robust function in `metafor` (Viechtbauer, 2010) to apply a cluster-robust adjustment of the model coefficients' variancecovariance matrix with a sandwich-type estimator to account for our small sample of studies (Hedges et al., 2010).

***Handling Variability in Effect Sizes Across Studies***

The average weighted correlation between the HME and children's math achievement was calculated using a random effects model, which assumes that rather than a single universal effect size, a distribution of potential effect sizes from different populations exists (Borenstein et al., 2009). A random effects model was chosen because the inconsistent methodology and definitional criteria used in HME research likely contribute to the high variability in effect sizes found across studies. In addition, many of the studies included in the present analysis were conducted across a variety of different settings, including different continents, making it reasonable to assume that differences between studies represent true differences among different populations that extend beyond sampling error.

***Heterogeneity of Effect Sizes***

In order to support the choice of a random effects model, we statistically evaluated the existence of heterogeneity by conducting a $Q$ test and calculating an unweighted sample-based estimate of $I^2$ (Higgins & Thompson, 2002), a descriptive statistic indicating the proportion of variance in effect sizes, from 0 to 100%, that is attributable to heterogeneity.

***Accounting for Dependent Effect Sizes***[2]

In order to statistically account for the reporting of more than one effect size per study sample a three-level correlated effects model was conducted. Studies and effect sizes were weighted based on correlated effects (rho or ρ), by clustering dependent effect sizes by both a

[2] Because the majority of our studies reported multiple effect sizes, we removed two pre-registered analyses from the present manuscript. First, we did not conduct a random effects meta-analysis that did not account for study dependence. Second, we did not create a trim-and-fill plot to assess publication bias because a trim and fill plot based on a multilevel object could not be created.

study-level and observation-level control variable, allowing for the estimation of unbiased standard error estimates. By modeling between-cluster (study) and within-cluster (effect size) heterogeneity separately, two variances were estimated, with $\sigma^2_1$ capturing the true variance between studies and $\sigma^2_2$ capturing the true variance within studies (Borenstein et al., 2009; Konstantopoulos, 2011). These separate estimates allowed us to evaluate the extent to which effect size differences are driven by between versus within study differences (Konstantopoulos, 2011).

**Analyzing Variability in Effect Sizes.** Then, to determine whether the observed heterogeneity in effect sizes was due to hypothesized moderators, multiple mixed effects models that controlled for study- and observation-level variance were tested using a meta-ANOVA framework. Specifically, we conducted a separate omnibus test for each moderator variable based on the $F$-distribution with $m$ (number of coefficients tested) and $k – p$ ($k$ = number of effect sizes, $p$ = number of model coefficients) degrees of freedom in order to determine whether subgroup effect sizes were statistically significantly different from one another in each moderator. In the event of a significant $F$-test, we evaluated effect size differences between subgroups of the moderator using pairwise comparisons for every possible subgroup pair. All pairwise comparisons were based on exploratory hypotheses and conducted based on the $t$-statistic with $k – p$ degrees of freedom for each subgroup of the moderator. The robust function was also used for all moderator analyses. We also applied the Benjamini-Hochberg correction to all pairwise comparisons in order to minimize the false discovery rate (Benjamini & Hochberg, 1995). Finally, we calculated overall effect sizes for each subgroup within a moderator by conducting random-effects multi-level models for each subgroup using datasets that only contained effect sizes for the specific subgroup.

**Evaluation of Publication Bias**

Publication bias, which has become an increasing problem in the psychological sciences, refers to the increased likelihood of studies with significant findings to be published and of studies with non-significant findings to be filed away in a drawer (i.e., the "file-drawer problem"; Rosenthal, 1979). Publication bias may lead to the estimation of a meta-analytic effect size that is smaller (or larger) than the true population effect size. Thus, in order to evaluate whether the average weighted correlation between the HME and children's math achievement calculated for the current meta-analysis showed evidence of publication bias, the `metafor` package (Viechtbauer, 2010) for `R version 3.5.3` (R Core Team, 2020) was used to conduct multiple tests of publication bias using both visual and statistical techniques.

***P-curve Analysis***

A *p*-curve analysis was also conducted to determine if there was evidence of *p*-hacking, the phenomenon where researchers collect or select data or modify statistical analyses until non-significant results become significant in order to increase their chances of publication (Head et al., 2015). Importantly, evidence of *p*-hacking typically indicates that a file-drawer problem exists because authors are likely to resort to *p*-hacking in order to obtain significant results so they can get their results published, while other researchers that do not *p*-hack and have non-significant findings are likely to be rejected for publication and filed away. We used *p*-curve analyses to determine the potential existence of publication bias by examining the distribution of significant *p*-values that corresponded to our observed effect sizes. *P*-curve analyses start with the calculation of *pp*-values, which represent the probability of obtaining each *p*-value if the null hypothesis were true (i.e., no significant effect) and are then summed to derive a $\chi^2$ value for testing the significance of the *p*-curve skew. A flat *p*-curve indicates that the probability of observing all *p*-values is uniform, and a right-skewed *p*-curve indicates that the effect is likely to be real, and the probability of lower *p*-values is greater than high *p*-values. Both of these scenarios likely point to a low chance of publication bias. However, a left-skewed *p*-curve shows evidence of *p*-hacking and indicates that the probability of high *p*-values is greater than the probability of low *p*-values. *P*-curve calculations were the only analyses not conducted in R and were instead conducted within the *p*-curve application available at: http://www.p-curve.com/app4/, which provided both binomial and continuous tests for publication bias and *p*-hacking.

Contrary to our pre-registration, rather than conducting an additional *p*-curve analysis using the R script from Simonsohn et al. (2014), we only conducted one *p*-curve analysis using the *p*-curve application in order to eliminate redundant analyses. In the same vein, rather than also conducting *p*-uniform calculations to test for publication bias, publication bias was assessed visually with a funnel plot and then parametrically using Egger's test (Egger et al., 1997).

***Funnel Plot***

Visually, publication bias was assessed using a funnel plot, a kind of scatter plot that visually depicts effect sizes relative to their standard errors (Sterne & Egger, 2001), which accounted for dependent effect sizes. A symmetrical distribution of observed effect sizes around the vertical line would indicate no publication bias, whereas an asymmetrical distribution would suggest potential publication bias. Symmetry was parametrically determined using an Egger's test, a meta-regression analysis that estimates effect size precision (i.e., the inverse sample size) as a predictor of the correlation coefficient, in a multi-level model (Egger et al., 1997). A statistically significant Egger's test indicates an asymmetrical distribution around the true population effect size, supporting the existence of publication bias. A non-significant Egger's test supports a symmetrical effect size distribution and a lack of evidence for publication bias.

## Sensitivity Analyses

Exploratory sensitivity analyses were conducted to statistically test the robustness of our meta-analytic results.

### *Fail-Safe N*

The Fail-safe *N* (Orwin, 1983) calculates the number of studies with null results (i.e., statistically non-significant Pearson correlation coefficients) that would have to be added to our observed outcomes to reduce the combined significance level to a target alpha level. Specifically, we tested alphas of $p > .05$ and $p > .01$ using the Rosenthal method, which has been nicknamed the "file drawer analysis" (Rosenthal, 1979).

### *Excluding a Potentially Influential Study*

Given that a single study in the final sample of studies reported 228 effect sizes (Cheung, 2013), which most likely had an inordinate influence on our meta-analytic results, we conducted the overall meta-analysis twice, once while excluding the study from the average weighted correlation calculation (reported in the main results) and once with the study included (reported in the supplemental materials). Because the Cheung (2012) study had not been added to our study sample at the time of the preregistration, excluding the single study from analyses was not preregistered.

# Results

## Final Article Sample

Our article searches yielded 1725 articles for review and coding. Only two articles from the final article sample were not captured by database searches and were procured from manual searching (Cai, 2003; Silinskas et al., 2010). During the first review of articles, 1192 of the articles were rejected based on titles or being duplicates, 431 more articles were excluded based on reviewing abstracts, and 57 were rejected based on reviewing methods or full manuscripts, resulting in a sample of 45 articles. After rejecting articles based on titles that indicated that the association of interest was not included in the study (i.e., the article measured the home learning environment but did not include an achievement measure, the article was a review or a qualitative study, or the article measured children's reading instead of math achievement), the most common reason for article exclusion was the use of a home environment and/or achievement measure that was not math-specific. Notably, a few recent prominent articles investigating the HME-achievement link through parent math anxiety, math applications at home, and/or parent homework help were excluded due to not reporting correlations for parent math anxiety and/or parent homework help with children's math achievement (Berkowitz et al., 2015; Gunderson et al., 2018; Maloney et al., 2015; Schaeffer et al., 2018).

A secondary article search was conducted in January 2020 to seek out additional grey literature, which was requested by reviewers and thus not preregistered; it yielded 42 additional articles for review. A total of 16 articles were eliminated for either not including a math-specific home learning or achievement measure ($k$ = 10), not reporting a correlation between an HME component and children's math achievement ($k$ = 3), or for including spontaneous number focusing as the child outcome measure ($k$ = 3), yielding 71 articles total.

Corresponding authors were contacted for 9 articles from the overall article sample that reported partial correlations between the HME and children's math achievement, controlling for factors such as age and socioeconomic status. However, 6 articles were excluded because authors no longer had access to the study data and could not calculate correlations again without covariates, or because corresponding authors did not respond to provide all the requested data (all 6 articles were from the original search: Blevins-Knabe & Musun-Miller, 1996; Dearing et al., 2012; Gunderson & Levine, 2011; Pezdek et al., 2002; Skwarchuk, 2009; Zippert & Ramani, 2017). Full correlations were provided for 3 of the 9 articles (Benavides-Varela et al., 2016; Mutaf-Yildiz et al., 2018a; Zippert & Rittle-Johnson, 2020).

The final sample reported in the present manuscript consisted of 631 effect size estimates, drawn from 68 distinct samples, reported in 64 manuscripts/studies. Each study contributed between 1 and 48 effect size estimates (median = 6). With the Cheung (2012) article included, there were 859 effect sizes drawn from 71 distinct samples reported in 65 manuscripts/studies. The entire article selection process is outlined in Figure 1.

**Overall Average Weighted Correlation between the Home Math Environment and Children's Math Achievement**

The results from the three-level correlated effects analysis yielded an average weighted correlation between the HME and math achievement of $r = .13$ (95% CI: [.09, .17], $p < .001$). Significant unexplained variance was found across the range of effect sizes included in the calculation of the overall average effect sizes ($Q$ [630] = 4947.56, $p < .001$, $I^2_{Total} = 95.15\%$), validating our use of a random-effects model for effect sizes that come from different populations. Approximately 95% of the variance in the correlation between HME and math achievement was not attributable to sampling error, with 60.81% of the unexplained variance due to between-study differences ($I^2_{Level2}$), 34.34% of the unexplained variance due to within-study differences ($I^2_{Level3}$), and the remaining 4.85% of the unexplained variance was due to sampling error. Next, multiple moderator analyses were conducted, one moderator at a time, in order to determine the assessment, study, and sample characteristics that may have significantly contributed to effect size heterogeneity.

## Moderation Effects of HME Assessment, Math Assessment, Study, and Sample Characteristics

All moderators were entered as categorical variables, except for age and parent education, which were entered as continuous. Table 2 shows the unstandardized beta values, 95% confidence intervals, the number of studies (*k*), and the number of effect sizes (*n*) for each moderator analysis. All univariate moderator analysis results showed that, even after accounting for the contribution of each moderator to effect size heterogeneity, a significant amount of unexplained heterogeneity remained in the overall effect size. Benjamini-Hochberg adjusted cutoffs were used to interpret the *p*-values obtained from all pairwise comparisons for subgroups of statistically significant moderators, and only comparisons that remained statistically significant after applying this correction were reported as such.

## Moderation Effects of HME Assessment Characteristics

A forest plot containing effect sizes, sample sizes, and 95% confidence intervals for each HME assessment moderator and subgroup is presented in Figure 2. All pairwise comparison results for HME assessment characteristics are presented in Table 2.

### *HME Component*

The omnibus test was statistically significant ($F[6, 55] = 2.43$, $p = .037$, $\sigma^2_1 = .02$, $\sigma^2_2 = .01$, $n = 619$, $k = 62$, $I^2 = 85.15\%$), indicating that at least one of the subgroups within the HME component moderator variable is statistically significantly different from at least one of the other subgroups. Pairwise comparisons indicated the average weighted correlation between the HME and children's math achievement for HME measures of indirect activities was statistically significantly higher than spatial activities (*b* = -.10, *t*[6] = -4.52, $p < .001$) and lower than math expectations (*b* = -.11, *t*[6] = -3.06, *p* = .004). Pairwise comparisons also indicated that the average weighted correlation between the HME and children's math achievement for HME measures of combined direct and indirect activities was statistically significantly higher than math talk (*b* = -.25, *t*[3] = -3.91, $p < .001$) and that math expectations were statistically significantly higher than spatial activities (*b* = -.20, *t*[3] = -2.83, *p* = .011).

***HME Measurement Method***

The overall omnibus test for HME measurement method was statistically significant (*F*[3, 59] = 5.40, *p* = . 002, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, *n* = 619, *k* = 63, $I^2$ = 86.78%), indicating that at least one of the subgroups within the HME measurement method moderator variable is statistically significantly different from at least one of the other subgroups. Pairwise comparisons indicated that the average weighted correlation between the HME and children's math achievement for HME measures that use frequency-based scales (*b* = .08, *t*[3] = 2.99, *p* = .005) or rating scales (*b* = .12, *t*[3] = 3.64, *p* = .001) was statistically significantly higher than HME measures that used checklists.

When the two-level measurement method moderator was tested, the subgroups included questionnaire-based measures (*r* = .14, 95% CI [.10, .18], *n* = 470, *k* = 50) and observation-based measures (*r* = .05, 95% CI [-.10, .21], *n* = 151, *k* = 15). The overall omnibus test was not statistically significant (*F*[1,61] = 2.11, *p* = .152, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, *n* = 621, *k* = 63, $I^2$ = 87.31%) indicating that there were no statistically significant differences in the average weighted correlation between the HME and children's math achievement based on whether the HME was assessed using questionnaire-based or observation-based measures.

***HME Score Calculation Method***

The overall omnibus test was not statistically significant $F[2, 58] = 1.64$, $p = .202$, $\sigma^2{}_1 = .02$, $\sigma^2{}_2 = .01$, $n = 602$, $k = 61$, $I^2 = 87.73\%$), indicating that there were no statistically significant differences in the average weighted correlation between the HME and children's math achievement based on whether the HME score was calculated using a latent factor score, sum score or a single item.

**Moderator Analyses for Math Assessment Characteristics**

A forest plot containing effect sizes, sample sizes, and 95% confidence intervals for each math assessment moderator is presented in Figure 3. All pairwise comparison results for math assessment characteristics are presented in Table 2.

***Math Domain***

The overall omnibus test was statistically significant ($F[3, 60] = 3.37$, $p = .024$, $\sigma^2{}_1 = .02$, $\sigma^2{}_2 = .01$, $n = 625$, $k = 64$, $I^2 = 87.16\%$ ), indicating that at least one of the subgroups within the math domain moderator variable is statistically significantly different from at least one of the other subgroups. Pairwise comparisons indicated that the average weighted correlation between the HME and children's math achievement for math assessments that measured the relations domain were statistically significantly lower than math assessments that measured multiple domains ($b = .08$, $t[3] = 3.77$, $p < .001$).

***Symbolic***

The overall omnibus test was statistically significant ($F[2, 60] = 5.97$, $p = .004$, $\sigma^2{}_1 = .02$, $\sigma^2{}_2 = .01$, $n = 630$, $k = 63$, $I^2 = 87.29\%$ ) indicating that at least one of the subgroups within the symbolic math assessment moderator variable is statistically significantly different from at least one of the other subgroups. However, when a Benjamini-Hochberg correction was applied, no pairwise comparisons were statistically significant.

***Composite***

The omnibus test for the composite math assessment moderator was statistically significant ($F$[1, 61] = 5.19, $p$ = .026, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, $n$ = 630, $k$ = 63, $I^2$ = 86.59%), indicating that the average weighted correlation between the HME and children's math achievement was statistically significantly higher for single math assessments than composite math assessments ($b$ = -0.04, $t$[1] = -2.28, $p$ = .026). However, when a Benjamini-Hochberg correction was applied, no pairwise comparisons were statistically significant.

***Standardized***

The omnibus test for the standardized math assessment moderator variable was statistically significant ($F$[1, 59] = 6.89, $p$ = .011, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, $n$ = 614, $k$ = 61, $I^2$ = 86.870%), indicating that the average weighted correlation between the HME and children's math achievement was statistically significantly lower for standardized versus unstandardized math assessments ($b$ = -0.05, $t$[1] = -2.62, $p$ = .011).

**Moderator Analyses for Study Characteristics**

A forest plot containing effect sizes, sample sizes, and 95% confidence intervals for the study characteristic moderator and its subgroups are presented in Figure 4.

The omnibus test for the longitudinal or concurrent moderator variable was not statistically significant ($F$[1, 62] = 3.30, $p$ = .074, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, $n$ = 631, $k$ = 64, $\mathrm{I}^2$ = 87.27%), indicating that studies reporting longitudinal effects were statistically no different from studies reporting concurrent effects ($b$ = 0.04, $t$[1] = 1.82, $p$ = .074).

**Moderator Analyses for Sample Characteristics**

A forest plot containing effect sizes, sample sizes, and 95% confidence intervals for each categorical sample characteristic moderator and subgroup is presented in Figure 5. All pairwise comparison results for sample characteristics are presented in Table 2.

***Age***

Age was not a statistically significant moderator of the average weighted correlation between the HME and children's math achievement ($F[1, 53] = 0.76$, $p = .387$, $\sigma^2_1 = .02$, $\sigma^2_2 = .01$, $n = 592$, $k = 55$, $I^2 = 76.37\%$). For every 1-year increase in age, the correlation between HME and children's math achievement decreases by .01, but this change is not statistically significant ($b = -0.01$, $t(1) = -0.87$, $p = .387$).

***Grade***

The overall omnibus test was statistically significant ($F[1, 62] = 6.11$, $p = .016$, $\sigma^2_1 = .02$, $\sigma^2_2 = .01$, $n = 631$, k = 64, $I^2 = 86.90\%$), indicating that the average weighted correlation between the HME and children's math achievement was statistically significantly higher for preschool/kindergarten samples than for primary/secondary samples ($b = -.06$, $t[1] = -2.47$, $p = .016$).

***Socioeconomic Status (SES)***

The overall omnibus test was not statistically significant ($F[2, 52] = 2.39$, $p = .101$, $\sigma^2_1 = .02$, $\sigma^2_2 = .01$, $n = 548$, $k = 55$, $I^2 = 88.34\%$), indicating that that there were no statistically significant differences in the average weighted correlation between the HME and children's math achievement based on the SES of the sample.

***Parent Education***

Parent education was not a statistically significant moderator of the average weighted correlation between the HME and children's math achievement ($F[1, 27] = 0.58$, $p = .454$, $\sigma^2_1 =$

.01, $\sigma^2_2$ = .01, *n* = 384, *k* = 29, $I^2$ = 88.72%). For every one percent increase in the percentage of parents within a sample who had attained any post-secondary education, the correlation between HME and children's math achievement decreases by 0.002, but this change was not statistically significant (*b* = -.002, *t*[1] = -0.76, *p* = .454).

## Exploratory Interaction Analyses

After our planned moderator analyses, we also conducted a series of exploratory multilevel correlated effects meta-regression models that included interaction terms in order to probe potential interactions revealed in the HME literature that also included at least one main effect that emerged as statistically significant in our univariate moderator analyses. The details and results of these interaction analyses are presented in the supplementary materials.

## Publication Bias

### *Funnel Plot*

First, publication bias was assessed using a funnel plot of effect sizes (x-axis) to standard errors (y-axis), which accounted for dependencies, and is depicted in Figure 6. A visual inspection showed that most of the estimates, both below and above the mean, are clustered near the top half of the funnel, suggesting high precision in effect size estimates overall. However, there are multiple studies outside of the shaded areas for the 90% (white), 95% (light grey), and 99% (dark grey) confidence intervals, suggesting that publication bias is possible. The Egger's test of funnel plot asymmetry was not statistically significant (*b* = -.47, *t*[630] = -.25, *p* = .802), indicating the funnel plot is not asymmetrical. In observing the plotted values, there appears to be slightly more effect sizes below the average weighted correlation coefficient (*r* = .13) rather than above this coefficient. Given this unexpected asymmetry, in that more published effect sizes were closer to a correlation of zero rather than above our study's estimated correlation of *r* = .13, the results do not support the existence of a file-drawer problem (wherein small effect sizes fail to be published or reported). Instead, it appears that the effect sizes reported in published studies are capturing the true effect size between the HME and children's math achievement and are not just skewed toward statistically significant results.

### *Publication Type Moderator*

As an additional exploratory step to test for publication bias, we ran one final correlated effects model with publication type as the moderator. The subgroups for testing whether publication type was a moderator included published peer-reviewed studies ($r$ = .12, 95% CI [.07, .17], $k$ = 52, $n$ = 529), unpublished studies, including theses, dissertations, and conference talks and posters ($r$ = .14, 95% CI [.08, .20], $k$ = 8, $n$ = 81), and studies from manuscripts in preparation or under review ($r$ = .13, 95% CI [-.03, .30], $k$ = 5, $n$ = 21). There were no statistically significant differences in the relations between the HME and children's math achievement based on whether a study was published, unpublished, or in preparation or under review ($F$ [2, 61] = 0.67, $p$ = .518, $\sigma^2_1$ = .02, $\sigma^2_2$ = .01, $n$ = 631, $k$ = 64). This further indicates that publication bias in the HME-math achievement research area is not likely to be an issue.

### *P-curve Analysis*

The $p$-curve analysis plot is presented in Figure 7. Results from the continuous $p$-curve analysis showed that both the full ($Z$ = -6.96, $p$ < .001) and half ($Z$ = -6.69, $p$ < .001) $p$-curve tests supported the existence of a significant right skew. These combination test results, which have been shown to be more robust to $p$-hacking than a simple $p$-curve test (Simonsohn et al., 2014), indicated that the set of significant findings had evidential value. This means that the results are likely not driven by the selective reporting of statistically significant analyses and/or studies. Furthermore, full $p$-curve, and both the half p-curve and binomial 33% power test were non-significant (full: $Z = 6.61$, p > .999; half: $Z = 6.90$, $p > .999$; binomial: $p > .999$), indicating that the *p*-curve does not support that the evidential value is inadequate nor absent. These combined results indicate that the present meta-analytic sample of studies has evidential value and does not show evidence of *p*-hacking.

## Sensitivity Analyses

### *Fail-Safe N*

According to the results of the Fail-safe *N* test using the Rosenthal approach, in order to achieve null population results (i.e., $r = .00$), an additional 286,100 effect sizes with null results (i.e., showing no statistically significant association between the HME and children's math achievement) are needed to achieve the target null *p*-value of > .05. To achieve a $p > .01$, an additional 142,713 effect sizes with null results ($r = .00$) are needed. These results show that our sample of effect sizes is likely capturing a true relation that is statistically significantly different from zero.

### *Excluding a Potentially Influential Study*

One study in our sample included 228 effect sizes (Cheung, 2012), which may have exercised an inordinate influence on our overall results. We conducted a sensitivity analysis to see if our results would change if this study was included in the average weighted correlation calculation. Results from the multilevel correlated effects meta-analysis that included Cheung (2012) demonstrated an average weighted correlation between the HME and children's math achievement that was the same as the average estimate that included all studies ($\Delta r = .00$; $r = .13$, 95% CI [.09, .17], $\sigma^2_1 = .02$, $\sigma^2_1 = .01$, $n = 861$, $k = 66$).

## Discussion

Individual differences in math achievement appear prior to formal schooling and tend to persist once schooling begins. Thus, finding early points of entry for children’s math skill development is important to reduce math achievement gaps between children. One potential early influence on children’s math achievement is the home math environment (HME), which represents a setting where social learning occurs between children and their caregivers (Vygotsky, 1978). There are many factors at play that can influence the nature of the association between the HME and children’s math achievement (Bronfenbrenner, 1979), including the specificity of the input children experience based on their ability (Bornstein, 2002). This diversity is reflected in the literature, as the association between the HME and children’s math achievement has been found to vary widely between studies, with correlations ranging from small to large and positive to negative. Furthermore, there is little to no standardization in HME measurement across studies, with definitions of the HME including a range of components like math-related activities (that directly and/or indirectly target math skills), parent math attitudes and/or beliefs, parent expectations for their children’s math achievement, any combination of the three, or parent math talk. There is a general lack of consensus on the role of the HME in children’s math achievement, lack of consistency in how the HME is measured, and parent math talk and other more traditional HME components tend to be studied separately. To address these inconsistencies, we conducted the present preregistered meta-analysis to synthesize a previously disjointed research area and calculate the average weighted correlation between the HME and children’s math achievement. Additionally, we conducted a series of moderator analyses to empirically test the impact of assessment, study, and sample characteristics on the magnitude of the associations found.

Overall, the results of the present meta-analysis showed that the home math environment and children's math achievement had an average weighted correlation that is small and positive. However, a correlation of $r$ = .13 translates to only 2% common variance between the two domains. As such, it appears that, when evaluating across a combined sample that includes many empirical studies on the HME and children's math achievement, the overall role of the HME in children's math achievement is consistent but minimal. Nonetheless, our tests for residual heterogeneity and moderation showed that there was a lot of variability in the association between the HME and children's math achievement, with effect size estimates among moderator subgroups varying widely.

## Significant Moderators and Interactions

Overall, our results showed that effect size variability cannot be attributed to a single source. Instead, the factors influencing the strength of the HME-math achievement link represent diverse sources of heterogeneity, capturing differences in HME and math assessment characteristics and sample characteristics too. This diversity in the significant moderators supports the role of Vygotsky's ZPD, Bronfenbrenner's ecological systems theory, and Bornstein's specificity principle in the relation between the HME and children's math achievement, demonstrating that children's individual characteristics (e.g. the grade they are in) interact with specific developmentally-appropriate environmental inputs (e.g., the HME component measured) to influence specific outcomes (e.g., children's math performance on standardized tests) through parent-child math-related interaction. Our findings highlight the importance of accounting for and considering many study design factors when measuring the association between the HME and children's math achievement.

### *HME Component and HME Measurement Method*

Based on the lack of standardization in how the HME is defined and measured across the HME-math achievement literature, we expected the HME component assessed to be driving the different correlations reported between the HME and children's math achievement, and our expectations were partially supported. Although we did not find any difference in the HME-math achievement relation based on whether math-related activities in the home were directly or indirectly targeting math skills, our HME Component x Grade meta-regression analysis showed that compared to primary/secondary children, the correlations were much higher for direct activities for preschool/kindergarten samples ($r$ = .18 vs. .02). Based on these results, it appears that parent engagement in direct math teaching, like using math flashcards, is most effective in the early years before formal schooling in the primary grades, and informal math learning experiences are more valuable for children in primary and secondary grades. One potential explanation for this is that preschool and kindergarten children are still developing foundational math knowledge that they can best acquire through explicit math teaching, whereas children in primary and secondary grades benefit more from the chance to apply their existing math knowledge to more real-world number-related experiences. This result is an example of the importance of the chronosystem-level influence of timing and is contrary to a different meta-analysis which showed that for preschool and kindergarten children, effect sizes that included indirect numeracy experiences were almost two times larger than those involving direct activities (Dunst et al, 2017). One potential reason for the difference in our meta-analytic results is our inclusion of studies that had a wider span of ages, with children ranging from 3.54 to 13.75 years old. Based on the current results, parents may do best to focus on direct math teaching when their children are in preschool and kindergarten, and then transition to incorporating more real-world applications for their primary and secondary grade children's math knowledge.

We also found that when the HME component included in the study was indirect math activities, the average weighted correlations was six times larger than when spatial activities was included as the HME component (*r* = .12 vs. .02). This small association for spatial activities is not surprising based on the specificity principle, which posits that specific spatial activities would be best suited to help develop children's spatial skills rather than their broader math skills. However, it is also promising for children's broad math achievement to discover that even activities outside the specific math domain can still partially support children's math development.

Finally, we found it interesting that the when the HME component included in a study was combined (direct and indirect) HME activities, there was a higher magnitude association with children's math achievement, compared to when parent math talk was the HME component included (*r* = .20 vs. .03). One potential explanation for this difference might be differences in measurement. We expected observation-based measures like those used to capture parent math talk to differ from questionnaire-based assessments due to the influence of social desirability bias and inaccurate recall in parent reports compared to a measure completed by an outside observer. However, when we tested this directly in a different moderation test, we did not find significant differences between the two measurement methods. This suggests that the differences in the correlation between the HME and children's math achievement when HME is measured as combined direct and indirect math activities compared to parent math talk is likely not because of different assessment methods used. In contrast to Missall et al. (2017), this may indicate that self-report measures are a valid and useful way to assess the HME, and future work may consider questionnaire-based approaches as a simple yet effective way to measure the HME when time and other resources are low. In fact, when we assessed the influence of a variety of different HME assessment methods on the association between the HME and children's math achievement, the only prominent finding was lower associations for effect sizes that included checklists compared to frequency-based scales and rating scales. This provides evidence that the common method of using number game checklists to serve as a proxy for all indirect math exposure in the home is probably not ideal, and researchers would do best to use the same kind of frequency- or rating-based scales typically employed for measuring direct activities. This may be related to the fact that checklists do not fully capture the routineness of a math-related interaction and may not be meeting the criteria for proximal processes to support skill development, namely the need for math-related interactions to be frequent, regular, and involve the child as an active participant.

Notably, our meta-regression models for HME Component x Age and HME Component x Grade showed that the relation between parent math talk and children's math achievement increases as children get older, and parent math talk is more strongly associated with primary/secondary grade children's math achievement ($r$ = .15 vs. .02, and not statistically significant for preschool/kindergarten). This may be at least partially attributable to the parents using specific math language that is more aligned with their children's math needs and abilities as children get older. This is supported by Son & Morrison's (2010) findings about the general home learning environment showing that parents tend to improve the home learning environment (including general language) as children approach school entry. Based on our findings, parents should focus on their use of math-related language for their primary and secondary grade children.

Overall, our HME component analyses revealed the importance of parental engagement in shared math (and even spatial) activities with their children to support math development, but we also found a vital role for parent socio-emotional factors in the growth of children's math skills. Specifically, we found partial support for our hypothesis that parent math expectations would play a pronounced role in children's math achievement compared to other forms of parent involvement. The association between parent math expectations and children's math achievement was almost twice as large as the association with indirect activities and eleven times as large as the association with spatial activities. This aligns with previous meta-analytic work by Fan and Chen (2001), which found that parent expectations stand out as the type of parent involvement with the strongest link to children's academic achievement and extends these results to a math-specific context.

Our HME Component x Age meta-regression analysis also revealed that the function of parent math expectations in their children's math achievement becomes increasingly more important as children get older. Some studies have shown that, compared to older children, younger children tend to be more influenced by parental values and experience more parent involvement (Eisenberg & Wolchik, 1992; Stevenson & Baker, 1987), but like our current meta-analytic results, previous meta-analyses have supported the importance of parents' academic expectations for both primary and secondary school children's academic achievement (Jeynes 2005, 2007). This may be because the effect of parent math expectations is not fully graspable by younger children, who are still focused on learning math basics. The influence of parent math expectations may need to build over time when children have a more fixed sense of their academic strengths and weaknesses (House, 1995) and are confident enough in their math abilities to feel like they can work to meet their parents' math expectations. Parents may also simply have higher expectations for the math achievement of their older children. Regardless of the mechanisms that explain this developmental pattern of an increasing association between parent math expectations and children's math achievement over time, this finding highlights the importance of incorporating math expectations for children at home, especially as they get older. Parents should most likely focus on setting high expectations for their children's math achievement, even as they progress through higher grades where math becomes more challenging.

**Math Assessment Moderator and Interaction Results**

Based on the wide array of math assessments used across the HME research literature, and meta-analytic evidence that the HME-achievement link varies depending on the math outcome assessed for preschool and kindergarten children (Dunst et al., 2017), we also probed the moderating effect of math domain and the effect of the Math Domain x Age interaction on the relation between the HME and children's math achievement. Based on our univariate analysis that did not account for age, we found that associations with the HME that included math assessments capturing multiple math domains were about twice as high as those that assessed the relations domain (.12 vs. .07). However, our Math Domain x Age meta-regression results revealed that the influence of different math outcomes changes over time. When children's age is equal to 0, multiple math domains have the highest effect size (.15), and as children get older, the numerical relations domain (which starts off with an $r$ = .07) becomes less important, with a .03 unit decrease in the correlation between the HME and numerical relations performance for every additional year of age, compared to a .002 unit increase for the arithmetic operations domain. These findings were different from the results reported in the meta-analysis by Dunst et al. (2017) on preschool/kindergarten children, which did not include a separate category for the relations domain, and instead found the lowest magnitude correlations ($r$ = .20) for “simple” math outcomes, which were what they called the domain capturing numbering abilities, like counting and number recognition.

These results provide two important insights. Although the idea of children who are 0 years old does not make practical sense, the finding that multiple math domains show the highest magnitude associations with the HME for very young children could be an indication that, rather than zeroing in on a single domain, HME investigations that include young children should assess multiple math domains. Given that children’s math skills grow in parallel (Van de Rijt & Van Luit, 1999), this could be attributable to the fact that very young children are still building foundational math skills and do not have enough expertise in any one area to sufficiently capture the full scope of their math ability and the HME’s role in it with a single domain assessment. Secondly, the twenty-fold yearly decrease in the association of the relations domain with the HME compared to the arithmetic operations domain, provides further evidence that parents adjust how they help their children based on how their math skills develop over time (Silinskas et al., 2010). It is important for parents to work on more complex math domains as their children’s math knowledge progresses in order to appropriately target their zones of proximal development and continue moving children’s math skills forward. Because we included only basic math domains due to the majority of HME research including preschool and kindergarten samples, future work should investigate whether or not the HME plays a role in more complex math domains for older samples.

Our Math Domain x HME Component meta-regression analysis revealed that of all four math domain subcomponents tested, numbering was the only math domain that was statistically significantly associated with every HME component. The only other significant association found was for arithmetic operations and direct activities, but correlations between direct math activities and the numbering domain were two and a half times higher than the associations with arithmetic operations. We also found that associations for the numbering domain, which includes skills like counting, number recognition, and cardinality, were stronger for behavior-related HME components (i.e., direct activities, indirect activities, and a combination of direct and indirect activities) than for the parent socio-emotional factors of attitudes and/or beliefs (.20, .15, and .16 vs. .06). Taken together, this shows us that home-based math activities play an important role in children's numbering skill development and that parent math attitudes and/or beliefs play a less pronounced but still significant role in children's numbering development. This may be explained by the fact that parents are more likely to focus on engaging in math teaching activities that focus on basic numerical concepts, like numbering, which serve as the foundation for more advanced number concepts, like arithmetic operations (Ramani et al., 2015) because children who struggle to master basic number concepts are the ones who are more likely to experience difficulty and receive parent support (Blevins-Knabe & Musun-Miller, 1996). This provides promising evidence for the potential for the HME to support children's math achievement. In fact, longitudinal work with preschoolers shows that initial level of math performance, as well as its growth, are best predicted by counting ability (Aunola et al., 2004). Thus, by supporting children's numbering skill development, the HME may also be fundamental for children's initial math performance and growth into more advanced domains as well.

The univariate moderator analyses also revealed that standardized math measures had higher magnitude correlations than unstandardized math measures, indicating that it may be wise to include standardized math assessments in future HME investigations in order to attain the most robust HME-achievement relations. This finding is contrasts from the common meta-analytic finding from across content areas that researcher-created outcome measures demonstrate significantly larger effect sizes (0.20–0.29 standard deviations greater; Cheung & Slavin, 2016; de Boer et al., 2014; Li & Ma, 2010) than assessments made by independent parties (like standardized tests). Although our results for the standardized moderator were unexpected, they were actually quite promising because they show that the HME may especially benefit children’s performance on high-stakes tests, which are vital to children’s advancement through school.

**Sample Characteristic Findings**

Finally, although our univariate moderator results did not support our hypothesis that there would be magnitude differences in the correlation between the HME and children’s math achievement between low- and high-SES samples, our meta-regression analysis for SES x HME Component interaction revealed that SES-driven differences do exist in the HME-math achievement link. One of the reasons we expected differences in the HME-achievement link based on SES is that research shows that children from low-income backgrounds who have lower levels of math achievement also tend to be engaged in less complex number activities at home (Saxe et al., 1987). Thus, we expected differences in the kinds of HME socialization and exposure present in high versus low SES households to be driving magnitude differences in the HME-achievement link, if they did exist. Overall, our meta-analysis would have benefitted from having more studies with high- and/or low-SES samples to enable us to make more definitive conclusions about the role of SES in the association between the HME and children's math achievement. Nevertheless, the recent meta-analysis using only preschool/kindergarten samples also failed to find a statistically significant correlation with home numeracy experiences for parent education and family SES (Dunst et al., 2017), and a previous meta-analysis on the association of the home literacy environment with children’s reading outcomes echoed this non-significant SES moderation finding (Bus et al., 1995).

For the SES x HME Component meta-regression analysis we found that direct activities had the strongest association with math achievement of all possible HME components for low-SES children but were weakest for high SES children. This is in line with Silinskas et al. (2010), which found that lower SES parents reported more teaching of math (and reading) than high SES parents, which may translate to a stronger correlation between direct activities and the HME. This may be attributable to the exosystem-level influence of low-SES parents having to hold more jobs or work more hours to make ends meet, leading low-SES parents to focus the time they do have on direct math teaching (Tracey & Young, 2002). For high-SES samples, one of the highest magnitude correlations with children's math achievement was found for a combination of direct and indirect math activities. This may be because high-SES parents have more time and resources to dedicate to both kinds of math learning. Additionally, parent math expectations were especially strongly associated with children's math achievement for high-SES parents. Previous work has found that lower SES parents are more likely to underestimate their children's math skill potential (DeFlorio & Beliakoff, 2015), which may lead them to hold lower expectations for their children's math performance compared to high-SES parents. Since the accuracy of parents' math expectations is a unique predictor of children's math achievement above age and SES (DeFlorio & Beliakoff, 2015), it may be that high-SES parents' more accurate math expectations are allowing for a stronger link between the HME and children's math achievement. Accordingly, rather than focusing on providing low-SES parents with guidelines about which math teaching activities to engage in at home, interventions would do well to focus on providing lower SES parents with more accurate knowledge about early math development (Starkey & Klein, 2000) to allow parents to understand and tap into their children's ZPD to push their children's math development forward.

Finally, for average SES samples, math talk had the strongest relations with math performance but was negatively associated with low SES children's math. Ramani et al. (2015) conducted a study in a low-SES sample investigating how both frequency of number activities and the quality of these interactions, which was measured by the amount and type of math talk used while engaging in activities, was affected by SES. They found that overall low-SES parents' use of math talk was low. They also found that although direct activities were sufficient for the development of foundational math knowledge, the quality of these direct math teaching experiences in terms of the types of talk caregivers provided was essential to the development of children's advanced number concepts. Low-SES parents have been shown to typically engage in talk about basic math concepts like counting and numeral identification (Levine et al., 2010), with infrequent engagement in talk about more advanced math concepts (Gunderson & Levine, 2011), and this was supported in the low-SES sample included in the study by Ramani et al. (2015). This could be an indication that the quality of math talk is lower for low-SES samples than higher SES samples. Thus, low-SES parents may not be engaging in the right kind of math talk to appropriately challenge their children within their ZPD and help them learn new concepts as their math skills advance. Given that the correlational association is not directional, the negative association between math talk and children's math achievement for low-SES samples could also be an indication that parents are engaging in more math talk because their children are struggling in math and need the additional support.

### *Grade*

The association between the HME and children's math achievement somewhat followed a developmental pattern of decreasing magnitude over time (Bus et al., 1995). Specifically, our results showed no significant moderation by age, but grade was a significant moderator, with younger (i.e., preschool/kindergarten) samples showing a stronger association between the HME and children's math achievement than primary/secondary grade samples. This aligns with the pattern typically found in syntheses on parental involvement, with parent assistance becoming less helpful as children get older and start to seek greater autonomy (Bronstein, 2015). The fact that the age moderator was not significant may be due to our age moderator being based on the average age of a sample, which led to a loss of variability and precision in the age moderator that would have helped us to pick up on age-related nuances in the HME-achievement link. Also, the HME research area in general is composed of mostly preschool and kindergarten samples, so there were likely not enough older samples to be representative, and more work should be done to examine the role of the HME in older children's math achievement as children advance into more complex math concepts. Additionally, longitudinal research focusing on wider age and/or grade ranges would provide more insight into whether there is a sensitive period for the HME and as to whether the potential effects of the HME fade out over time.

**Is "More" HME Better than "Less" HME?**

Across all moderators, our expectation of magnitude differences was driven by the reasoning that having "more" HME would translate into a more dedicated focus on math in the home and thus, a stronger association between the HME and children's math achievement. This expectation that "more is better" was based on the assumption that more HME would represent more parental resources and effort put forth into child-oriented activities and interactions overall, especially those related to math achievement (Grolnick & Slowiaczek, 1994). This parental focus on children's math development—in terms of more positive social interactions around math and an increased emphasis on math achievement expectations— may serve to convey the importance of math education (Epstein, 1988) and help children feel more competent in math (Patterson, 1986). In turn, this may create a stronger link between the HME and children's math achievement. By the same token, less frequent parent involvement in math and less emphasis on the HME would likely lead to a weaker link between the HME and children's math achievement due to the reduced emphasis on math education and less effort put forth to cultivate child competence in math. Looking beyond the domain-specific impacts of the HME, the increased stimulation in the home associated with "more" HME, would also be expected to promote the development of domain-general skills that support academic achievement overall, like positive attitudes toward learning and motivation (Grolnick & Slowiaczek, 1994).

Although positive parental involvement supports children's autonomy-building and helps children develop positive work orientations, which are vital for academic achievement (Bronstein et al., 2005), some parent involvement may be deleterious for children's math achievement. For example, in cases where parents are demanding, controlling, and use an authoritarian parenting style that fails to support children's autonomy-building, more involved parenting, in terms of "more HME," may actually be detrimental to children's math outcomes (Bronstein et al., 2005). Thus, although we expect the magnitude of the association to be higher with all forms of "more involved" parenting, the association between the HME and children's math achievement may be negative for certain parenting styles and positive for others. Future HME research should assess parenting styles and attitudes in order to examine how different parenting inputs may influence the HME-math achievement relation.

**Limitations**

The present work is not without limitations. Although meta-analysis, in general, enabled us to analyze the results of a large sample of aggregated studies on the HME-math achievement relation, increasing the power we have to draw statistically-driven conclusions, the results presented here were limited to the studies that have been conducted on the relation of interest. We were also limited to answering the research questions that the literature allowed. However, unexplained variance that remained in the accumulated effect sizes after accounting for all moderators tested indicates that there are other study factors we did not test that may also moderate the HME-math achievement relation, like parenting styles and quality (Bronstein et al., 2005), and future HME work should aim to assess more parenting quality measures to understand their effect on the HME-math achievement relation. Additionally, the present meta-analysis did not assess whether the quality of the included studies moderated the relation between the HME and children's math achievement. Given that meta-analyses are dependent on the quality of the studies they include due to their reliance on effect size parameters from studies that vary in the thoroughness of their methods and study designs (Gersten, et al., 2000), future work should examine this potential influence.

Another limitation of this work is we cannot speak to the causal direction underlying the HME-achievement link. More parent involvement in math-related interactions in the home may help support children's math achievement. On the other hand, children with higher math achievement may elicit more math-related interactions from their parents. Another possibility is that the HME-achievement link is bidirectional, with each exercising a significant influence on the other. Finally, it may be the case that there is no causal direction, but simply a correlational one due to common or confounding etiological influences (e.g., Hart et al., 2021). There are unique designs that might help untangle the correlational nature of the association (e.g., Erbeli et al., 2019), including longitudinal designs, in addition to experimental designs, and we encourage the field to move beyond simply measuring the correlation in the future.

Our search strategy and timing could have also presented some limitations. The search was done for articles available or published in the fall of 2018 with a follow-up grey literature call in January of 2020. There may have been studies that have been published since our last search or grey literature call that are missing from our study sample and could potentially impact our meta-analytic findings. There were also six studies that had to be excluded from our study sample because of reporting partial correlations that the authors were unable to or did not provide, which may have provided additional results that would have impacted our meta-analytic findings. Although the breadth of our search and the broadness of our search terms was likely effective in capturing a sample of studies that was ecologically valid, it is also important to keep in mind that some effect sizes may be missing that would deepen our understanding of the association between the HME and children's math achievement. Another factor that could limit the generalizability of our findings is the fact that most of our study sample came from western countries and more specifically, from North America. More international work is needed to ascertain how the role of the HME in children's math achievement may differ in non-western countries.

**Implications**

Overall, we found there was a statistically significant, yet small in magnitude correlation between the HME and children's math achievement. This finding has theoretical implications. We have previously discussed the specific findings and their theoretical implications, but broadly we have found support for Vygotsky's sociocultural learning theory, with the HME serving as an environment where math-specific social learning occurs through parents' participation in activities to directly and indirectly target children's math skills as well as their math-related attitudes, beliefs, expectations, and utterances. The specificity of this input will partially determine the extent to which certain math skills are supported, with math-specific inputs showing larger effects than inputs that target spatial skills. Specificity will also matter for the developmental time points at which parents' math-related inputs will function to push development forward by providing support within children's ZPD. For example, children in preschool and kindergarten are likely to show stronger associations with certain aspects of the HME than children in primary and secondary grades. Many interacting environmental influences will shape the nature and effects of these parent-child social learning interactions, from the influence of time and timing in the chronosystem to the influence of parents' expertise, resource availability, and orientation toward math via the exosystem, to parents emphasis on the importance of math achievement via the mesosystem influence of parent math expectations.

These findings also have practical implications. The low but significant average correlation, coupled with our lack of knowledge of the causal direction underlying the correlation, might make it seem like it is not worth it to design HME interventions to support children's early math achievement. However, this meta-analysis, like all correlational studies, only speak to "what was," not "what could be" via an intervention. Although post-intervention effect sizes were purposely excluded from the present study sample, the findings from the few intervention studies in this area showed that home-based math interventions can sometimes be effective to help parents become aware of their children's mathematical understandings (Muir, 2011), which is essential for parents to know which skills to work on at home. Although many intervention studies have required extensive parent training and often included the use of electronic tools that must be returned after the intervention (Berkowitz et al., 2015; Starkey & Klein, 2000; Van Tuijl et al., 2001; Niklas et al., 2016a), some low cost intervention studies that used researcher-created board games have found that playing linear board games that include numbers and counting up to 10 may help support children's critical linear numerical representational skills necessary for math development (Ramani & Siegler, 2008; Siegler & Ramani, 2009). However, when the researchers are not able to supervise the parents' teaching techniques and the store-bought game of Chutes and Ladders, which requires children to count as high as 100 is used, the board game playing intervention was not effective, with children in the control group advancing just as much as children in the intervention group, probably due to maturation (Sonneschein et al., 2016). Board game playing still appears to be a potentially promising avenue for effective intervention on parent-child math interactions and improving children's math skills in some scenarios if parents can be properly trained and developmentally-appropriate board games are employed. In the end, interventions focus on pushing up the mean, and thus an intervention to increase the quantity and quality of HME interactions might indeed have a direct impact on improving children's math achievement. There is considerable theoretical support for the role of parents in their children's learning, and this supports the usefulness of designing and testing HME interventions.

**Conclusion**

Whether the direction of the association is from children's math achievement to the HME or from the HME to children's math achievement, one over-arching conclusion can be drawn: the association between the two is small and positive. Overall, this work has both theoretical implications for theories explaining the HME and math achievement association and practical implications that will advise the future development of effective interventions.

**Table 1**

Article Coding Key

| Category | Value Description |
|---|---|
| HME Assessment Characteristics | |
| HME Component | 1 = Direct activities |
| | 2 = Indirect activities |
| | 3 = Combination direct & indirect activities |
| | 4 = Attitudes and/or beliefs |
| | 5 = Math expectations |
| | 6 = Spatial activities |
| | 7 = Math talk |
| HME Calculation | 1 = Latent factor score |
| | 2 = Sum score |
| | 3 = Single item |
| HME Measurement Method | 1 = Frequency-based scale |
| | 2 = Rating scale |
| | 3 = Checklist |
| | 4 = Observation |
| Math Assessment Characteristics | |
| Math Domain | 1 = Arithmetic operations |
| | 2 = (Numerical) Relations |
| | 3 = Numbering |
| | 4 = Multiple domains |
| Symbolic | 1 = Symbolic |
| | 2 = Non-symbolic |
| | 3 = Combination of symbolic & non-symbolic |
| Composite | 1 = Composite |
| | 2 = Single Measure |
| Standardized | 1 = Standardized |
| | 2 = Unstandardized |
| Study Characteristics | |
| Longitudinal | 1 = Longitudinal relation |
| | 2 = Concurrent relation |
| Sample Characteristics | |
| Grade | 1 = PK/KG |
| | 2 = Primary/Secondary |
| Socioeconomic Status (SES) | 1 = Low SES (50% or more) |
| | 2 = Average SES |
| | 3 = High SES (50% or more) |

*Note*. All preregistered moderator subgroups that are not included above were excluded because of having too few studies and too little variability for valid comparisons.

**Table 2**

*Univariate Pairwise Comparisons of HME Component, HME Measurement Method, Math Domain, Symbolic, Timed, Composite, Standardized, Longitudinal, and Grade*

| Moderator Variable | beta | 95% CI LL | 95% CI UL | *k* | *n* |
|---|---|---|---|---|---|
| HME Component | | | | | |
| Direct vs. Indirect | .01 | -.08 | .10 | 37 | 286 |
| Direct vs. Direct/Indirect | .06 | -.04 | .17 | 49 | 230 |
| Direct vs. Attitudes/Beliefs | -.03 | -.15 | .08 | 38 | 250 |
| Direct vs. Expectations | .05 | -.02 | .13 | 37 | 166 |
| Direct vs. Spatial Activities | .002 | -.003 | .01 | 34 | 146 |
| Direct vs. Math Talk | -.001 | -.23 | .23 | 34 | 151 |
| Indirect vs. Direct/Indirect | .001 | -.08 | .08 | 46 | 272 |
| Indirect vs. Attitudes/Beliefs | -.06† | -.11 | -.01 | 40 | 292 |
| **Indirect vs. Expectations** | **-.11*** | **-.19** | **-.04** | **37** | **208** |
| **Indirect vs. Spatial Activities** | **-.10**** | **-.15** | **-.05** | **31** | **188** |
| Indirect vs. Math Talk | -.09† | -.19 | -.0004 | 34 | 193 |
| Direct/Indirect vs. Attitudes/Beliefs | -.05† | -.10 | -.002 | 38 | 236 |
| Direct/Indirect vs. Expectations | -.04 | -.11 | .03 | 31 | 152 |
| Direct/Indirect vs. Spatial Activities | -.20† | -.35 | -.04 | 29 | 132 |
| **Direct/Indirect vs. Math Talk** | **-.25**** | **-.38** | **-.12** | **33** | **137** |
| Attitudes/Beliefs vs. Expectations | .03 | -.09 | .16 | 26 | 172 |
| Attitudes/Beliefs vs. Spatial Activities | -.04 | -.17 | .10 | 29 | 152 |
| Attitudes/Beliefs vs. Math Talk | .09 | -.21 | .39 | 27 | 157 |
| **Expectations vs. Spatial Activities** | **-.20*** | **-.34** | **-.05** | **21** | **68** |
| Expectations vs. Math Talk | -.29† | -.58 | -.01 | 20 | 73 |
| Spatial Activities vs. Math Talk | .03 | -.07 | .13 | 15 | 53 |
| HME Measurement Method | | | | | |
| Frequency-Based Scale vs. Rating Scale | -.03† | -.07 | -.001 | 49 | 465 |
| **Frequency-Based Scale vs. Checklist** | **.08*** | **.03** | **.14** | **47** | **339** |
| Frequency-Based Scale vs. Observation | -.12 | -.25 | .01 | 57 | 445 |
| **Rating Scale vs. Checklist** | **.12**** | **.05** | **.18** | **29** | **174** |
| Rating Scale vs. Observation | -.12 | -.27 | .03 | 39 | 280 |
| Checklist vs. Observation | -.06 | -.30 | .18 | 20 | 154 |
| Math Domain | | | | | |
| Arithmetic Operations vs. Relations | -.04† | -.07 | -.01 | 29 | 251 |
| Arithmetic Operations vs. Numbering | .02 | -.02 | .05 | 24 | 237 |
| Arithmetic Operations vs. Multiple | .01 | -.03 | .05 | 61 | 393 |
| Relations vs. Numbering | .03 | -.02 | .09 | 20 | 232 |
| **Relations vs. Multiple** | **.08**** | **.04** | **.12** | **57** | **388** |

| | | | | | |
|---|---|---|---|---|---|
| Numbering vs. Multiple Symbolic | .01 | -.04 | .06 | 55 | 374 |
| Symbolic vs. Non-symbolic | .005† | -.03 | .04 | 37 | 397 |
| Symbolic vs. Symb/Non-Symb | .06† | .02 | .11 | 60 | 458 |
| Non-Symbolic vs. Symb/Non-Symb | .05 | .01 | .08 | 52 | 405 |

*Note*. *$p \leq .01$, **$p \leq .001$, †predictor variables no longer statistically significant after applying a Benjamini-Hochberg correction with a false discovery rate of .05; beta = unstandardized regression coefficient; 95% CI = 95% confidence interval; LL = lower level of the confidence interval; UL = higher level of the confidence interval; *k* = number of studies, *n* = number of effect sizes; statistically significant pairwise comparisons are **bolded**; PK/KG = preschool and kindergarten samples.

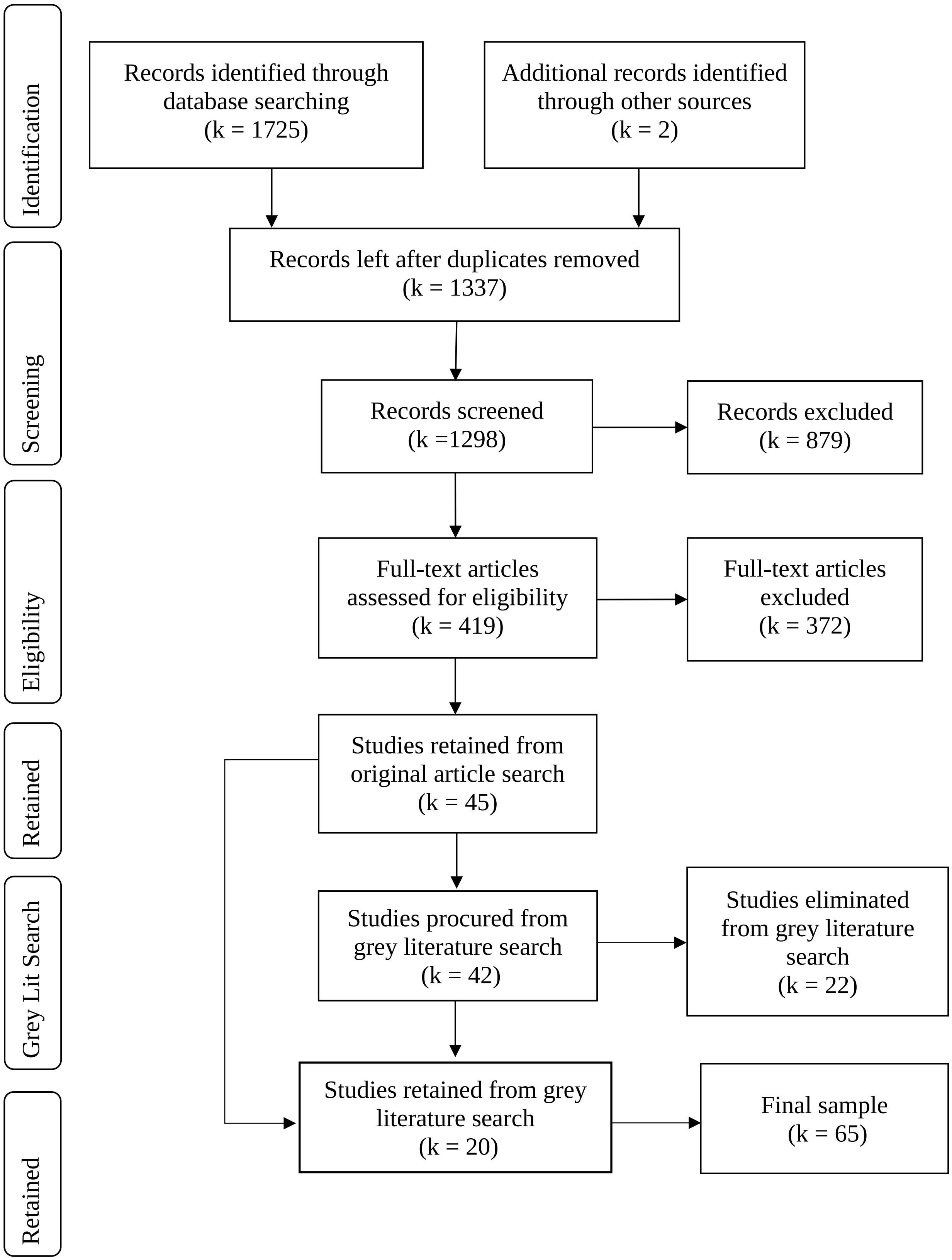


*Figure 1*. Article selection flow chart. The overall analysis only included 64 articles.

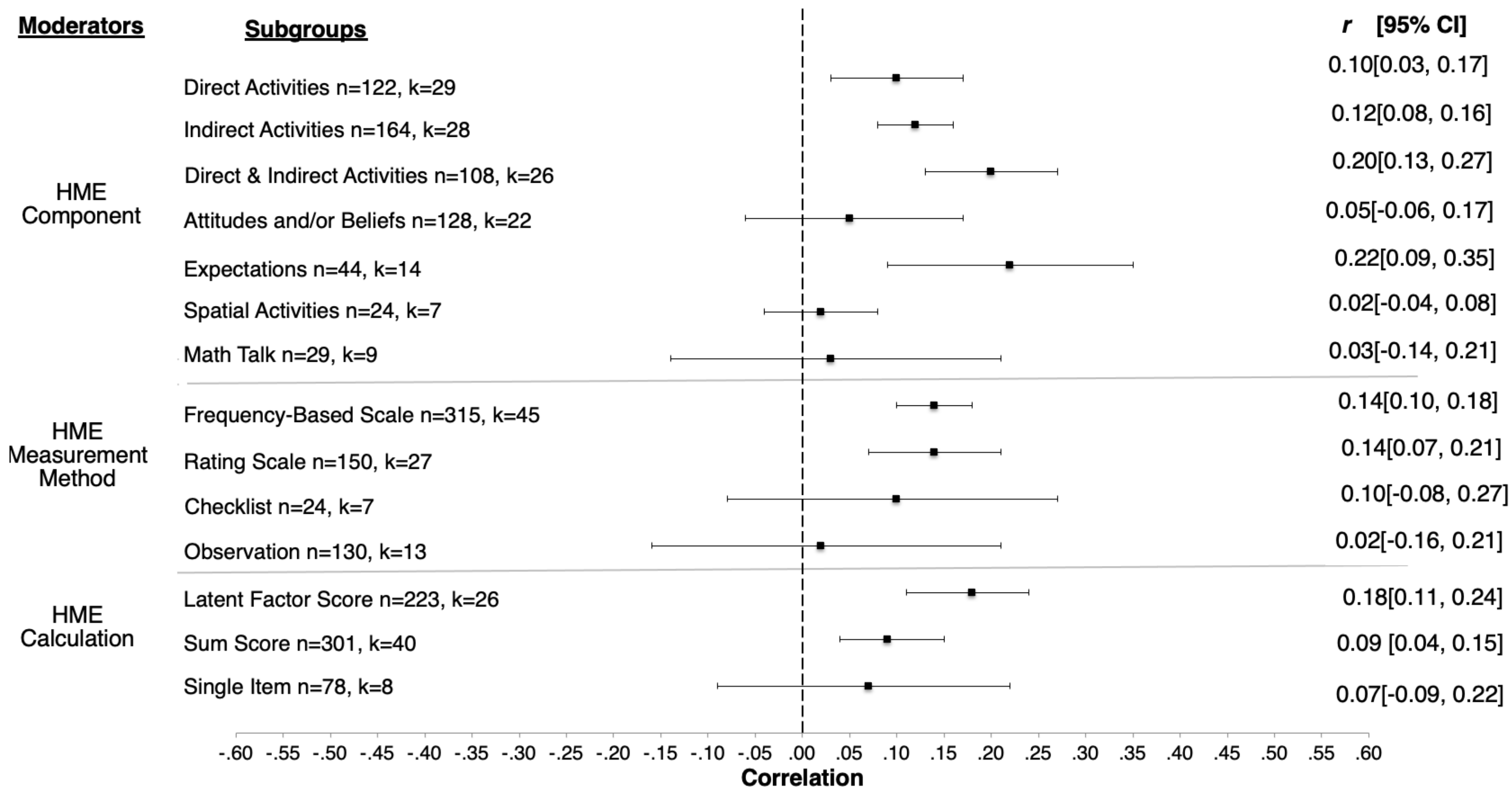


*Figure 2.* Effect size estimates (x-axis) and 95% confidence intervals (standard error bars) for HME assessment moderators, including HME component, HME measurement method, and HME calculation. The number of effect sizes (*n*) and studies (*k*) for each moderator subgroup is presented in the left column, and the correlation coefficient (*r*) and its corresponding 95% confidence interval (95% CI) are presented in the far-right column. Confidence intervals that cross 0 (dashed line) represent effect sizes that

are not significantly different from zero. All confidence intervals were calculated using robust variance estimation for small-sample correction.

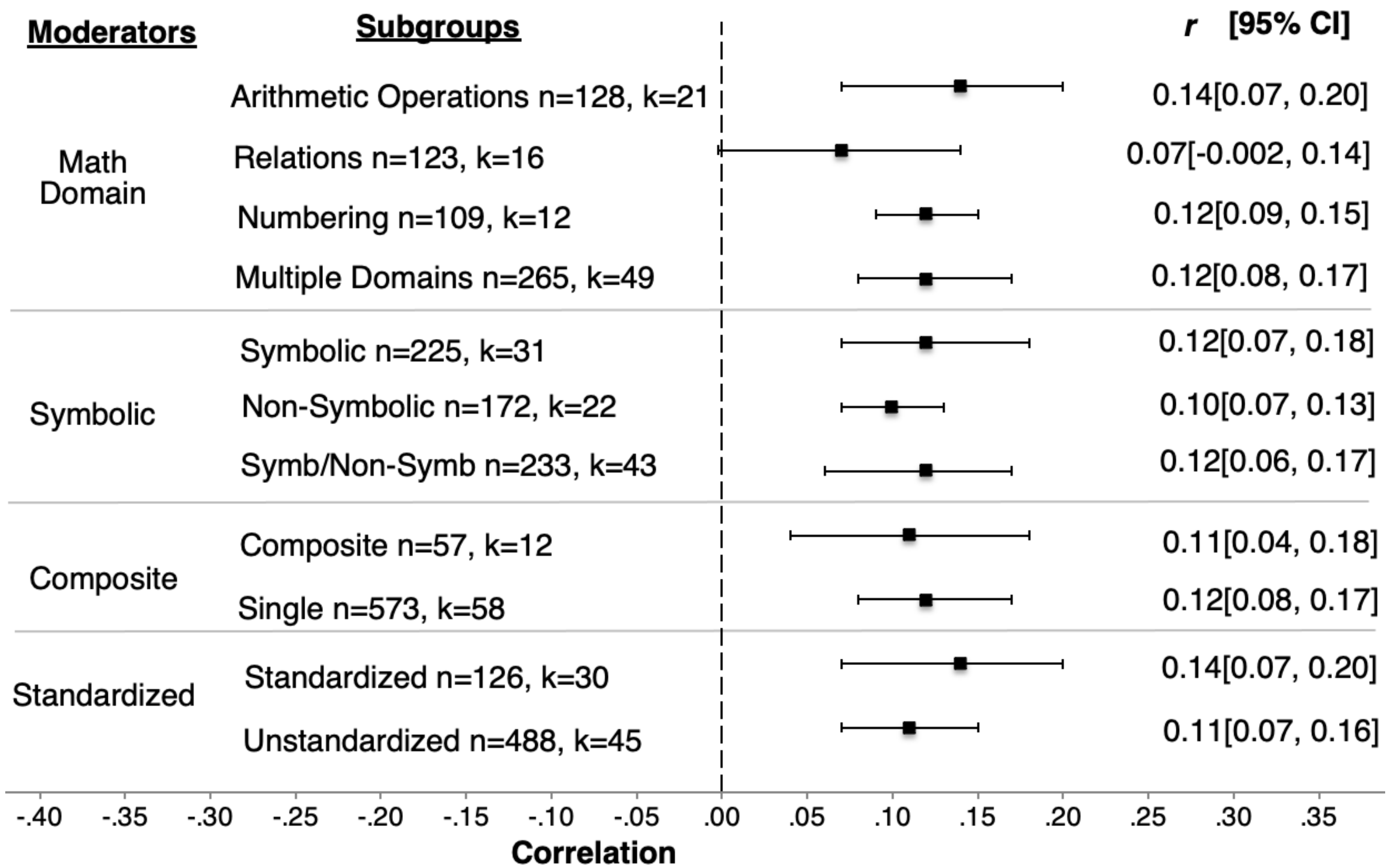


*Figure 3.* Effect size estimates (x-axis) and 95% confidence intervals (standard error bars) for math assessment moderators, including math domain, symbolic, composite, and standardized. The number of effect sizes (*n*) and studies (*k*) for each moderator subgroup is presented in the left column. Confidence intervals that cross 0 (dashed

line) represent effect sizes that are not significantly different from zero. Symb/Non-Symb = combination of symbolic and non-symbolic math assessments; All confidence intervals were calculated using robust variance estimation for small-sample correction.

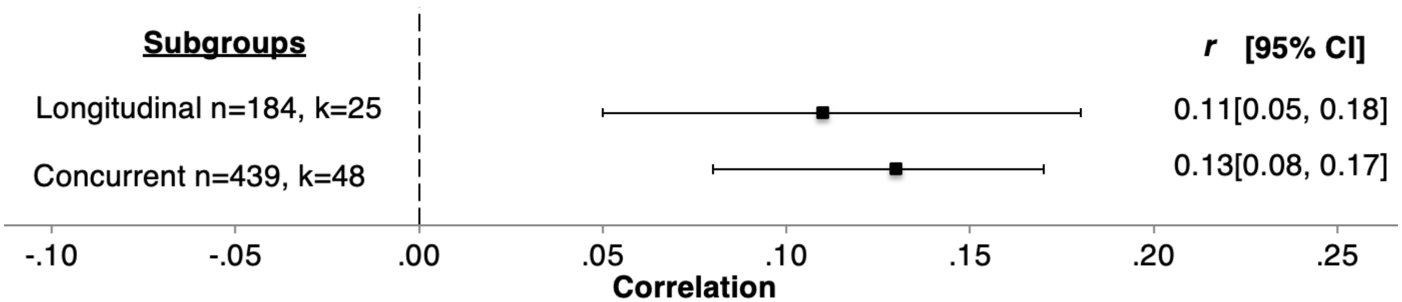


*Figure 4.* Effect size estimates (x-axis) and 95% confidence intervals (standard error bars) for the moderator capturing whether longitudinal or concurrent effect sizes were used. The number of effect sizes (*n*) and studies (*k*) for each moderator subgroup is presented in the left column. Confidence intervals that cross 0 (dashed line) represent effect sizes that are not significantly different from zero. All confidence intervals were calculated using robust variance estimation for small-sample correction.

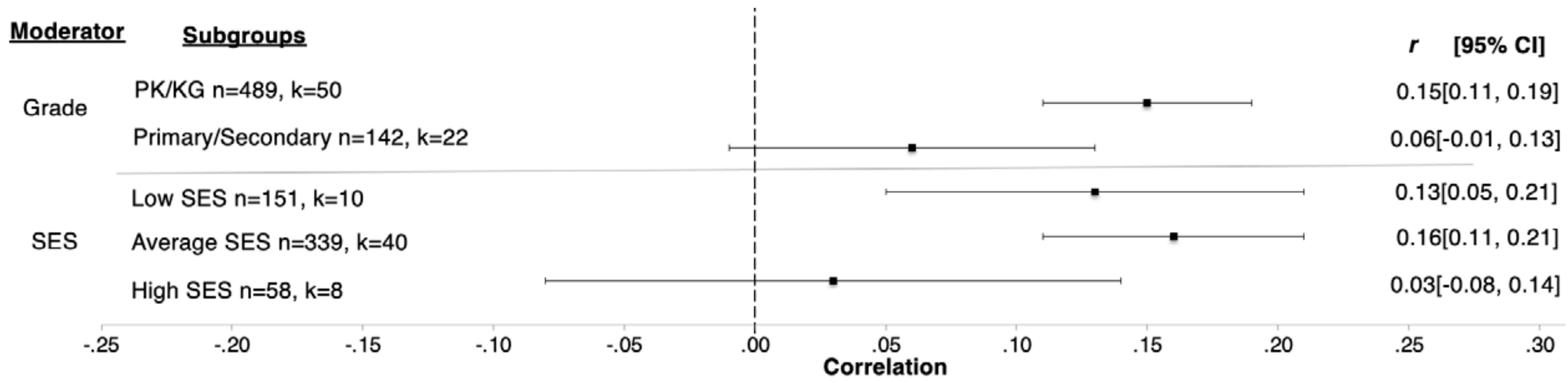


*Figure 5.* Effect size estimates (x-axis) and 95% confidence intervals (standard error bars) for sample characteristic moderators, including grade and SES. The number of effect sizes (*n*) and studies (*k*) for each moderator subgroup is presented to the left. Confidence intervals that cross 0 (dashed line) represent effect sizes that are not significantly different from zero. PK/KG = preschool and kindergarten samples; Primary/Secondary = samples that include primary grades (1-5) and secondary grades (6-12). All confidence intervals were calculated using robust variance estimation for small-sample correction.

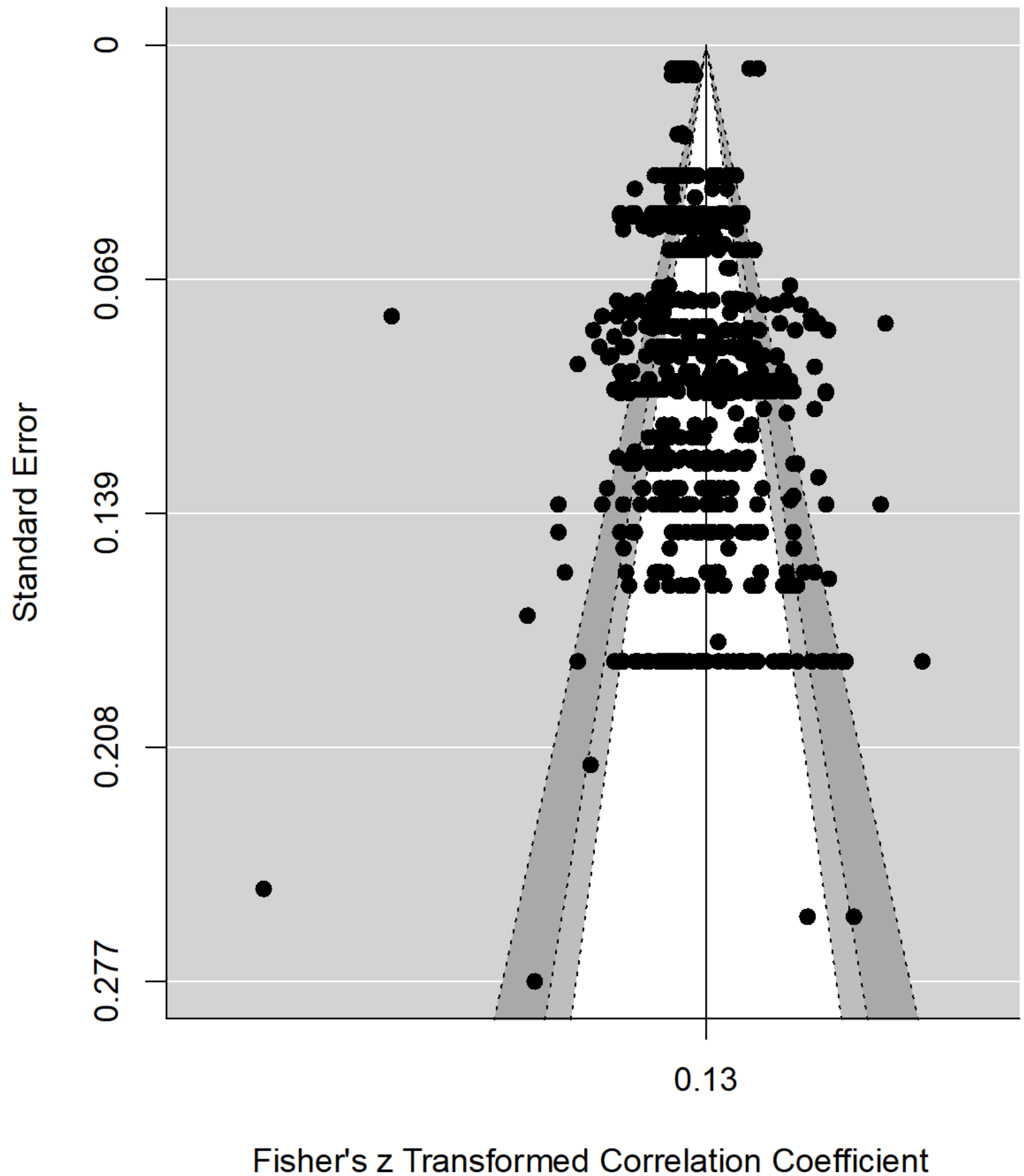


*Figure 6.* Enhanced funnel plot of the multilevel correlated effects meta-analysis results with the average weighted correlation of $r = .13$ and confidence intervals on the $90^{th}$ (white), $95^{th}$ (gray), and $99^{th}$ (dark gray) percentiles.

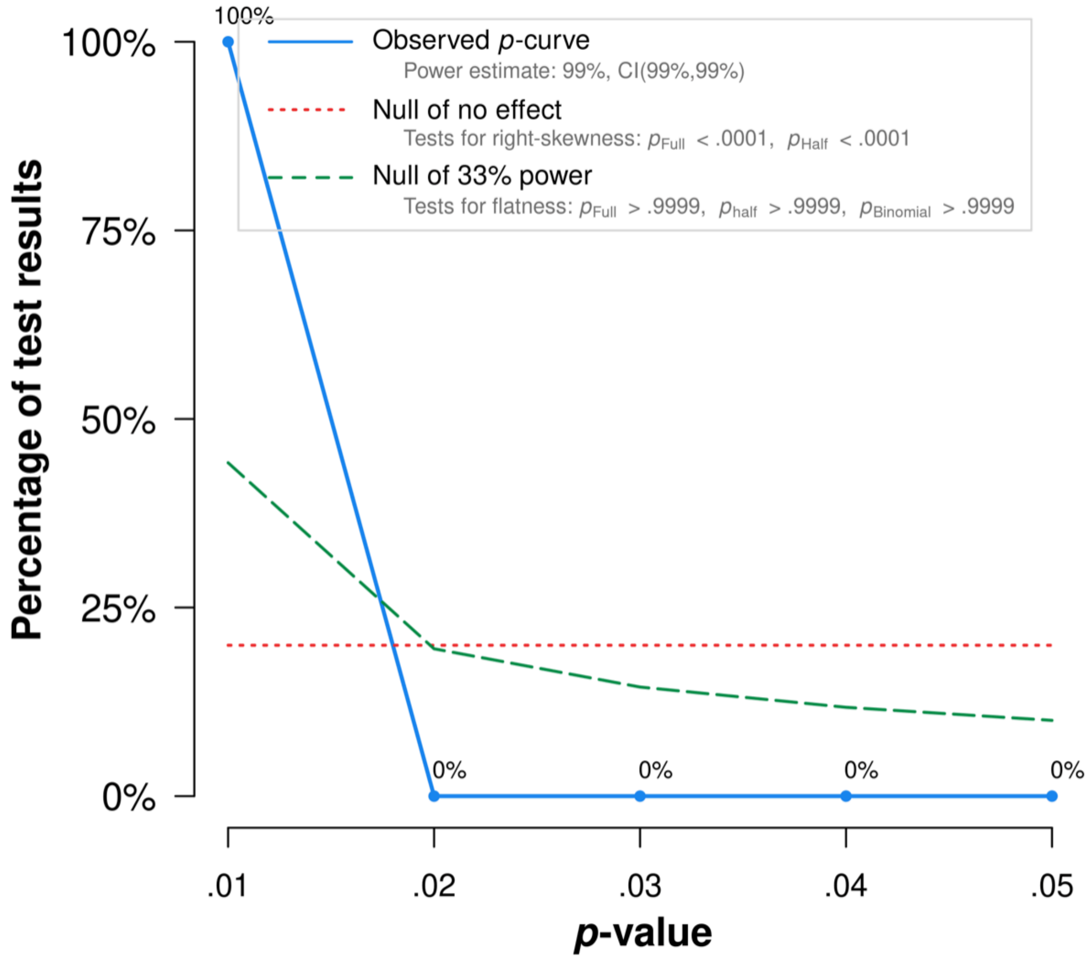


*Figure 7.* P-curve analysis results showing that a significant right skew and non-significant results at 33% power for both the full and half p-curve tests, indicating evidential value and a lack of *p*-hacking in our study sample.

**References**

Anderson, A. (1997). Families and mathematics: A study of parent–child interactions. *Journal for Research in Mathematics Education*, *28*, 484–511.

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, *96*(4), 699-713.

*Benavides-Varela, S., Butterworth, B., Burgio, F., Arcara, G., Lucangeli, D., & Semenza, C. (2016). Numerical activities and information learned at home link to the exact numeracy skills in 5–6 years-old children. *Frontiers in Psychology*, *7*, 94.

*Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly*, *19*(1), 4-20.

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society: Series B (Methodological)*, *57*(1), 289-300.

*Bennett, A. (2017). *Early numerical experiences* (Doctoral dissertation, Loughborough University).

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science*, *350*(6257), 196-198.

*Bhanot, R., & Jovanovic, J. (2005). Do parents' academic gender stereotypes influence whether they intrude on their children's homework?. *Sex Roles*, *52*(9-10), 597-607.

Bleeker, M. M., & Jacobs, J. E. (2004). Achievement in math and science: Do mothers' beliefs matter 12 years later?. *Journal of Educational Psychology*, *96*(1), 97-109.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting: An International Journal of Research and Practice*, *5*(1), 35-45.

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child Development and Care*, *165*(1), 41-58.

Borenstein, M., Cooper, H., Hedges, L., & Valentine, J. (2009). Effect sizes for continuous data. *The Handbook of Research Synthesis and Meta-Analysis*, *2*, 221-235.

Bornstein, M. H. (2002). Parenting infants. In M. H. Bornstein (Ed.), *Handbook of parenting Vol. 1 Children and Parenting* (2e, pp. 3-43). Mahwah, NJ: Lawrence Erlbaum Associates.

Bronfenbrenner, U. (1979). Contexts of child rearing. *American Psychologist, 34,* 844–858.

Bus, A. G., Van Ijzendoorn, M. H., & Pellegrini, A. D. (1995). Joint book reading makes for success in learning to read: A meta-analysis on intergenerational transmission of literacy. *Review of Educational Research*, *65*(1), 1-21.

*Cai, J. (2003). Investigating parental roles in students' learning of mathematics from a cross-national perspective. *Mathematics Education Research Journal*, *15*(2), 87-106.

*Cain-Caston, M. (1993). Parent and student attitudes toward mathematics as they relate to third grade mathematics achievement. *Journal of Instructional Psychology*, *20*(2), 96.

*Cankaya, Z. O. (2013). *The Role of Number Naming Systems and Numeracy Experiences in the*

*Development of Early Numeracy Knowledge* [Master’s Thesis, Carleton University].

Cantlon, J. F. (2012). Math, monkeys, and the developing brain. *Proceedings of the National Academy of Sciences*, *109*(Supplement 1), 10725-10732.

Casey, B. M., Dearing, E., Dulaney, A., Heyman, M., & Springer, R. (2014). Young girls’ spatial and arithmetic performance: The mediating role of maternal supportive interactions during joint spatial problem solving. *Early Childhood Research Quarterly*, *29*(4), 636-648.

Castro, M., Expósito-Casas, E., López-Martín, E., Lizasoain, L., Navarro-Asencio, E., & Gaviria, J. L. (2015). Parental involvement on student academic achievement: A meta-analysis. *Educational Research Review, 14*, 33-46.

Cheung, S. K. (2012). *Promoting Preschoolers' Number Sense and Interest in Learning Mathematics: How Should Parents Interact with Their Children During Home Numeracy Activities?* (Doctoral dissertation, Chinese University of Hong Kong).

*Cheung, S. K., Dulay, K. M., & McBride, C. (2020). Parents’ characteristics, the home environment, and children’s numeracy skills: How are they related in low-to middle-income families in the Philippines?. *Journal of Experimental Child Psychology, 192*, 104780.

Cheung, S. K., & Fong. (2019, June). Roles of parents’ sense of perfectionism towards children’s learning and home numeracy activities in young children’s numeracy skills. Paper presented at the meeting of the Mathematical Cognition and Learning Society, Carleton University, Canada.

Cheung, S. K., & McBride, C. (2017). Effectiveness of parent–child number board game playing in promoting Chinese kindergarteners' numeracy skills and mathematics interest. *Early Education and Development*, *28*(5), 572-589.

Cheung, A. C., & Slavin, R. E. (2016). How methodological features affect effect sizes in education. *Educational Researcher*, *45*(5), 283-292.

*Cheung, S. K., Yang, X., Dulay, K. M., & McBride, C. (2018). Family and individual variables associated with young Filipino children's numeracy interest and competence. *British Journal of Developmental Psychology*, *36*(2), 334-353.

*Ciping, D., Silinskas, G., Wei, W., & Georgiou, G. K. (2015). Cross-lagged relationships between home learning environment and academic achievement in Chinese. *Early Childhood Research Quarterly*, *33*, 12-20.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement*, *20*(1), 37-46.

Cohn, L. D., & Becker, B. J. (2003). How meta-analysis increases statistical power. *Psychological Methods*, *8*(3), 243-253.

Cohrssen, C., Church, A., & Tayler, C. (2014). Purposeful pauses: Teacher talk during early childhood mathematics activities. *International Journal of Early Years Education*, *22*(2), 169-183.

Davis-Kean, P. E. (2005). The influence of parent education and family income on child achievement: the indirect role of parental expectations and the home environment. *Journal of Family Psychology*, *19*(2), 294-304.

Dearing, E., Casey, B. M., Ganley, C. M., Tillinger, M., Laski, E., & Montecillo, C. (2012). Young girls' arithmetic and spatial skills: The distal and proximal roles of family socioeconomics and home learning experiences. *Early Childhood Research Quarterly*, *27*(3), 458-470.

de Boer, H., Donker, A., & van der Werf, M. (2014). Effects of the attributes of educational interventions on students' academic performance: A meta-analysis. *Review of Educational Research*, *84*(4), 509–545.

*De Keyser, L., Bakker, M., Rathé, S., Wijns, N., Torbeyns, J., Verschaffel, L., & De Smedt, B. (2020). No Association Between the Home Math Environment and Numerical and Patterning Skills in a Large and Diverse Sample of 5-to 6-year-olds. *Frontiers in Psychology*, *11*, 3238.

*DeFlorio, L. L. (2011). *The influence of the home learning environment on preschool children's informal mathematical development: Variation by age and socioeconomic status*(Doctoral dissertation, UC Berkeley).

DeFlorio, L., & Beliakoff, A. (2015). Socioeconomic status and preschoolers' mathematical knowledge: The contribution of home activities and parent beliefs. *Early Education and Development*, *26*(3), 319-341.

*del Río, M. F., Susperreguy, M. I., Strasser, K., & Salinas, V. (2017). Distinct influences of mothers and fathers on kindergartners' numeracy performance: The role of math anxiety, home numeracy practices, and numeracy expectations. *Early Education and Development*, *28*(8), 939-955.

Doig, B., & De Lemos, M. (2000). *I Can Do Maths. Teacher's Guide and Test Booklets Levels A and B.* ACER Press Customer Service, Australian Council for Educational Research, Private Bag 55, Camberwell, Victoria 3124, Australia.

*Drummond, K. V., & Stipek, D. (2004). Low-income parents' beliefs about their role in children's academic learning. *The elementary school journal*, *104*(3), 197-213.

Dunst, C. J., Hamby, D. W., Wilkie, H., & Dunst, K. S. (2017). Meta-analysis of the relationship between home and family experiences and young children's early numeracy learning. In *Engaging families as children's first mathematics educators*(pp. 105-125). Springer, Singapore.

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number word use. *British Journal of Developmental Psychology*, *4*(3), 269-288.

Eccles, J. S., Jacobs, J. E., & Harold, R. D. (1990). Gender role stereotypes, expectancy effects, and parents' socialization of gender differences. *Journal of Social Issues*, *46*(2), 183-201.

Egger, M., Smith, G. D., Schneider, M., & Minder, C. (1997). Bias in meta-analysis detected by a simple, graphical test. *Bmj*, *315*(7109), 629-634.

Eisenberg, N., Wolchik, S. A., Goldberg, L., & Engel, I. (1992). Parental values, reinforcement, and young children's prosocial behavior: A longitudinal study. *The Journal of Genetic Psychology*, *153*(1), 19-36.

*Elliott, L., Braham, E. J., & Libertus, M. E. (2017). Understanding sources of individual variability in parents' number talk with young children. *Journal of Experimental Child Psychology*, *159*, 1-15.

*Ellis, A. (in prep) The role of formal schooling on early number skills: Parent questionnaire data (Unpublished Doctoral Dissertation). University of Michigan, Ann Arbor, MI, USA.

*Else-Quest, N. M., Hyde, J. S., & Hejmadi, A. (2008). Mother and child emotions during mathematics homework. *Mathematical Thinking and Learning*, *10*(1), 5-35.

Epstein, C. F. (1988). *Deceptive distinctions: Sex, gender, and the social order*. New Haven & London: Yale University Press.

Erbeli, F., van Bergen, E., & Hart, S. A. (2020). Unraveling the relation between reading comprehension and print exposure. *Child Development*, *91*(5), 1548-1562.

*Esplin, J. A., Neilson, B. G., Austin, A. M. B., Blevins-Knabe, B., Hendershot, S. M., & Loesch, L. A. (2016). Number line skills and home numeracy activities for preschoolers in center-based and family-based childcare. In *Early childhood mathematics skill development in the home environment* (pp. 105-126). Springer, Cham.

Evans, M. A., & Shaw, D. (2008). Home grown for reading: Parental contributions to young children's emergent literacy and word recognition. *Canadian Psychology/Psychologie Canadienne*, *49*(2), 89-95.

Fan, X., & Chen, M. (2001). Parental involvement and students' academic achievement: A meta-analysis. *Educational Psychology Review*, *13*(1), 1-22.

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, *123*, 53-72.

Foster, M. E., Anthony, J. L., Clements, D. H., Sarama, J., & Williams, J. M. (2016). Improving mathematics learning of kindergarten students through computer-assisted instruction. *Journal for Research in Mathematics Education*, *47*(3), 206-232.

Galindo, C., & Sheldon, S. B. (2012). School and home connections and children's kindergarten achievement gains: The mediating role of family involvement. *Early Childhood Research Quarterly*, *27*(1), 90-103.

Gayán, J., & Olson, R. K. (2003). Genetic and environmental influences on individual differences in printed word recognition. *Journal of Experimental Child Psychology*, *84*(2), 97-123.

Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology*, *47*(2), 175-192.

Grolnick, W. S., & Slowiaczek, M. L. (1994). Parents' involvement in children's schooling: A multidimensional conceptualization and motivational model. *Child Development*, *65*(1), 237-252.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: relations between parents’ input and children’s cardinal-number knowledge. *Developmental Science*, *14*(5), 1021-1032.

Gunderson, E. A., Park, D., Maloney, E. A., Beilock, S. L., & Levine, S. C. (2018). Reciprocal relations among motivational frameworks, math anxiety, and math achievement in early elementary school. *Journal of Cognition and Development*, *19*(1), 21-46.

Gutman, L. M., & Midgley, C. (2000). The role of protective factors in supporting the academic achievement of poor African American students during the middle school transition. *Journal of Youth and Adolescence*, *29*(2), 223-249.

*Hart, S. A., Ganley, C. M., & Purpura, D. J. (2016). Understanding the home math environment and its role in predicting parent report of children’s math skills. *PloS one*, *11*(12), e0168227.

Hart, S. A., Little, C., & Van Bergen, E. (2021). Nurture might be nature: Cautionary tales and proposed solutions. *NPJ Science of Learning*, *6*(1), 1-12.

Hart, B., & Risley, T. R. (1995). *Meaningful differences in the everyday experiences of young American children.* Baltimore: Brookes.

Head, M. L., Holman, L., Lanfear, R., Kahn, A. T., & Jennions, M. D. (2015). The extent and consequences of p-hacking in science. *PLoS Biol*, *13*(3), e1002106.

Hedges, L. V., Tipton, E., & Johnson, M. C. (2010). Robust variance estimation in meta-regression with dependent effect size estimates. *Research Synthesis Methods*, *1*(1), 39-65.

Higgins, J. P., & Thompson, S. G. (2002). Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine*, *21*(11), 1539-1558.

Hindman, A. H., Skibbe, L. E., Miller, A., & Zimmerman, M. (2010). Ecological contexts and early learning: Contributions of child, family, and classroom factors during Head Start, to literacy and mathematics growth through first grade. *Early Childhood Research Quarterly*, *25*(2), 235-250.

Holod, A. (2012). *Parental Time or Money: What Matters More for Children's School Success?* (Doctoral dissertation, Columbia University).

House, J. D. (1995). The predictive relationship between academic self-concept, achievement expectancies, and grade performance in college calculus. *The Journal of Social Psychology*, *135*(1), 111-112.

*Huang, Q., Zhang, X., Liu, Y., Yang, W., & Song, Z. (2017). The contribution of parent–child numeracy activities to young Chinese children's mathematical ability. *British Journal of Educational Psychology*, *87*(3), 328-344.

Huntsinger, C. S., Jose, P. E., & Larson, S. L. (1998). Do parent practices to encourage academic competence influence the social adjustment of young European American and Chinese American children?. *Developmental Psychology*, *34*(4), 747-756.

Huntsinger, C. S., Jose, P. E., Larson, S. L., Balsink Krieg, D., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. *Journal of Educational Psychology*, *92*(4), 745-760.

*Huntsinger, C. S., Jose, P. E., & Luo, Z. (2016). Parental facilitation of early mathematics and reading skills and knowledge through encouragement of home-based activities. *Early Childhood Research Quarterly*, *37*, 1-15.

Jacobs, J. E., & Eccles, J. S. (1992). The impact of mothers' gender-role stereotypic beliefs on mothers' and children's ability perceptions. *Journal of Personality and Social Psychology*, *63*(6), 932-944.

Jameson, M. M. (2014). Contextual factors related to math anxiety in second-grade children. *The Journal of Experimental Education*, *82*(4), 518-536.

Jeynes, W. H. (2005). A meta-analysis of the relation of parental involvement to urban elementary school student academic achievement. *Urban education*, *40*(3), 237-269.

Jeynes, W. H. (2007). The relationship between parental involvement and urban secondary school student academic achievement: A meta-analysis. *Urban Education*, *42*(1), 82-110.

Keith, P. B., & Lichtman, M. V. (1994). Does parental involvement influence the academic achievement of Mexican-American eighth graders? Results from the National Education Longitudinal Study. *School Psychology Quarterly*, *9*(4), 256-272.

Kellaghan, T., Sloane, K., Alvarez, B., & Bloom, B. S. (1993). *The home environment and school learning: Promoting parental involvement in the education of children*. San Francisco, CA: Jossey-Bass.

*Kleemans, T., Segers, E., & Verhoeven, L. (2013). Relations between home numeracy experiences and basic calculation skills of children with and without specific language impairment. *Early Childhood Research Quarterly*, *28*(2), 415-423.

*Kleemans, T., Segers, E., & Verhoeven, L. (2018). Individual differences in basic arithmetic skills in children with and without developmental language disorder: Role of home numeracy experiences. *Early Childhood Research Quarterly*, *43*, 62-72.

*Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly*, *27*(3), 471-477.

Konstantopoulos, S. (2011). Fixed effects and variance components estimation in three-level meta-analysis. *Research Synthesis Methods*, *2*(1), 61-76.

Lareau, A. (1987). Social class differences in family-school relationships: The importance of cultural capital. *Sociology of Education*, *60*(2), 73-85.

*LeFevre, J. A., Xu, C., Gu, F., Douglas, H., Gardner, C., & Capparini, C. (2017, June). *How do children learn math? Relations between parents' math skill, home numeracy activities, and children's performance*. Talk presented at the 27th Annual Meeting of Canadian Society for Brain, Behaviour, and Cognitive Science at the University of Regina, Regina, CA.

*LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science/Revue canadienne des sciences du comportement*, *41*(2), 55-66.

*LeFevre, J. A., Polyzoi, E., Skwarchuk, S. L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children?. *International Journal of Early Years Education*, *18*(1), 55-70.

*Lehrl, S., Ebert, S., Blaurock, S., Rossbach, H. G., & Weinert, S. (2020). Long-term and domain-specific relations between the early years home learning environment and students' academic outcomes in secondary school. *School Effectiveness and School Improvement*, *31*(1), 102-124.

*Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge?. *Developmental Psychology*, *46*(5), 1309-1319.

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. Educational Psychology Review, 22(3), 215–243.

*Lindberg, S. M., Hyde, J. S., & Hirsch, L. M. (2008). Gender and mother-child interactions during mathematics homework: The importance of individual differences. *Merrill-Palmer Quarterly (1982-)*, 232-255.

Lipsey, M. W., & Wilson, D. B. (2001). *Practical meta-analysis*. SAGE publications, Inc.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, *17*(5), 714-726.

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational effects of parents' math anxiety on children's math achievement and anxiety. *Psychological Science*, *26*(9), 1480-1488.

*Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. *Early Childhood Research Quarterly*, *28*(4), 692-703.

*Missall, K., Hojnoski, R. L., Caskie, G. I., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development*, *26*(3), 356-376.

Missall, K. N., Hojnoski, R. L., & Moreano, G. (2017). Parent–child mathematical interactions: examining self-report and direct observation. *Early Child Development and Care*, *187*(12), 1896-1908.

*Mutaf-Yıldız, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2018a). Frequency of home numeracy activities is differentially related to basic number processing and calculation skills in kindergartners. *Frontiers in psychology*, *9*, 340.

*Mutaf-Yıldız, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2018b). Investigating the relationship between two home numeracy measures: A questionnaire and observations during Lego building and book reading. *British Journal of Developmental Psychology*, *36*(2), 354-370.

Mutaf-Yıldız, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2020). Probing the Relationship Between Home Numeracy and Children's Mathematical Skills: A Systematic Review. *Frontiers in Psychology*, *11*, 2074.

*Napoli, A. R., & Purpura, D. J. (2018). The home literacy and numeracy environment in preschool: Cross-domain relations of parent–child practices and child outcomes. *Journal of Experimental Child Psychology*, *166*, 581-603.

*Napoli, A. R., & Purpura, D. J. (in prep). *A text message-based home numeracy intervention for preschoolers and their parents*. Manuscript submitted for publication.

National Research Council. (2009). Mathematics learning in early childhood: Paths toward excellence and equity. Washington, DC: The National Academies Press.

*Niklas, F., Cohrssen, C., & Tayler, C. (2016a). Improving preschoolers’ numerical abilities by enhancing the home numeracy environment. *Early Education and Development*, *27*(3), 372-383.

*Niklas, F., Cohrssen, C., & Tayler, C. (2016b). Parents supporting learning: A non-intensive intervention supporting literacy and numeracy in the home learning environment. *International Journal of Early Years Education*, *24*(2), 121-142.

*Niklas, F., Nguyen, C., Cloney, D. S., Tayler, C., & Adams, R. (2016c). Self-report measures of the home learning environment in large scale research: Measurement properties and associations with key developmental outcomes. *Learning Environments Research*, *19*(2), 181-202.

*Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, *29*(3), 327-345.

Orwin, R. G. (1983). A fail-safe N for effect size in meta-analysis. *Journal of Educational Statistics*, *8*(2), 157-159.

Patall, E. A., Cooper, H., & Robinson, J. C. (2008). Parent involvement in homework: A research synthesis. *Review of Educational Research*, *78*(4), 1039-1101.

Patterson, G. R. (1986). Performance models for antisocial boys. *American Psychologist*, *41*(4), 432-444.

Peters, S. (1998). Playing games and learning mathematics: The results of two intervention studies. *International Journal of Early Years Education*, *6*(1), 49-58.

Pezdek, K., Berry, T., & Renno, P. A. (2002). Children's mathematics achievement: The role of parents' perceptions and their involvement in homework. *Journal of Educational Psychology*, *94*(4), 771-777.

*Phillipson, S., & Phillipson, S. N. (2007). Academic expectations, belief of ability, and involvement by parents as predictors of child achievement: A cross-cultural comparison. *Educational Psychology*, *27*(3), 329-348.

*Puccioni, J. (2015). Parents' conceptions of school readiness, transition practices, and children's academic achievement trajectories. *The Journal of Educational Research, 108*(2), 130-147.

Purpura, D. J., Baroody, A. J., & Lonigan, C. J. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology*, *105*(2), 453-464.

Purpura, D. J., & Ganley, C. M. (2014). Working memory and language: Skill-specific or domain-general relations to mathematics?. *Journal of Experimental Child Psychology*, *122*, 104-121.

*Purpura, D. J., King, Y. A., Rolan, E., Hornburg, C. B., Schmitt, S. A., Hart, S. A., & Ganley, C. M. (2020). Examining the factor structure of the home mathematics environment to delineate its role in predicting preschool numeracy, mathematical language, and spatial skills. *Frontiers in Psychology*, *11*, 1925.

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal*, *50*(1), 178-209.

Purpura, D. J., Logan, J. A., Hassinger-Das, B., & Napoli, A. R. (2017). Why do early mathematics skills predict later reading? The role of mathematical language. *Developmental Psychology*, *53*(9), 1633-1642.

Purpura, D. J., Schmitt, S. A., & Ganley, C. M. (2017). Foundations of mathematics and literacy: The role of executive functioning components. *Journal of Experimental Child Psychology*, *153*, 15-34.

R Core Team (2020). R: A language and environment for statistical computing.

*Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15-33.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*(2), 375-394.

Rosenthal, R. (1979). The file drawer problem and tolerance for null results. *Psychological Bulletin*, *86*(3), 638-641.

Saxe, G. B., Guberman, S. R., Gearhart, M., Gelman, R., Massey, C. M., & Rogoff, B. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development*, *52*(2), i-162.

Schaeffer, M. W., Rozek, C. S., Berkowitz, T., Levine, S. C., & Beilock, S. L. (2018). Disassociating the relation between parents' math anxiety and children's math achievement: Long-term effects of a math app intervention. *Journal of Experimental Psychology: General*, *147*(12), 1782-1790.

*Segers, E., Kleemans, T., & Verhoeven, L. (2015). Role of parent literacy and numeracy expectations and activities in predicting early numeracy skills. *Mathematical Thinking and Learning*, *17*(2-3), 219-236.

Sénéchal, M., & LeFevre, J. A. (2002). Parental involvement in the development of children's reading skill: A five-year longitudinal study. *Child Development*, *73*(2), 445-460.

Shalev, R. S., Auerbach, J., Manor, O. H. A. D., & Gross-Tsur, V. (2000). Developmental dyscalculia: prevalence and prognosis. *European Child & Adolescent Psychiatry*, *9*(2), S58-S64.

*Silinskas, G., Leppänen, U., Aunola, K., Parrila, R., & Nurmi, J. E. (2010). Predictors of mothers' and fathers' teaching of reading and mathematics during kindergarten and Grade 1. *Learning and Instruction*, *20*(1), 61-71.

Simonsohn, U., Nelson, L. D., & Simmons, J. P. (2014). p-curve and effect size: Correcting for publication bias using only significant results. *Perspectives on Psychological Science*, *9*(6), 666-681.

Simpkins, S. D., Fredricks, J. A., & Eccles, J. S. (2012). Charting the Eccles' expectancy-value model from mothers' beliefs in childhood to youths' activities in adolescence. *Developmental Psychology*, *48*(4), 1019-1032.

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home?. *Early Childhood Education Journal*, *37*(3), 189-197.

*Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63-84.

*Söchtig, I. & Niklas, F. (under review). Association of Home Numeracy Environment and Home Literacy Environment with child competencies. Submitted for a special issue in the Journal "Bildungsforschung".

Son, S. H., & Morrison, F. J. (2010). The nature and impact of changes in home learning environment on development of language and academic skills in preschool children. *Developmental Psychology*, *46*(5), 1103-1118.

*Soto-Calvo, E., Simmons, F. R., Adams, A. M., Francis, H. N., & Giofre, D. (2020). Pre-schoolers' home numeracy and home literacy experiences and their relationships with early number skills: Evidence from a UK study. *Early Education and Development*, *31*(1), 113-136.

Starkey, P., & Klein, A. (2000). Fostering parental support for children's mathematical development: An intervention with Head Start families. *Early Education and Development*, *11*(5), 659-680.

Starkey, P., & Klein, A. (2008). Sociocultural influences on young children's mathematical knowledge. *Contemporary Perspectives on Mathematics in Early Childhood Education*, 253-276.

Sterne, J. A., & Egger, M. (2001). Funnel plots for detecting bias in meta-analysis: guidelines on choice of axis. *Journal of Clinical Epidemiology*, *54*(10), 1046-1055.

Stevenson, D. L., & Baker, D. P. (1987). The family-school relation and the child's school performance. *Child Development*, *58*(5), 1348-1357.

*Susperreguy, M. I., & Davis-Kean, P. E. (2016). Maternal math talk in the home and math skills in preschool children. *Early Education and Development*, *27*(6), 841-857.

*Susperreguy, M. I., Davis-Kean, P. E., Duckworth, K., & Chen, M. (2018). Self-concept predicts academic achievement across levels of the achievement distribution: Domain specificity for math and reading. *Child Development*, *89*(6), 2196-2214.

*Susperreguy, M. I., Di Lonardo Burr, S., Xu, C., Douglas, H., & LeFevre, J. A. (2020a). Children's home numeracy environment predicts growth of their early mathematical skills in kindergarten. *Child Development*, *91*(5), 1663-1680.

*Susperreguy, M. I., Douglas, H., Xu, C., Molina-Rojas, N., & LeFevre, J. A. (2020b). Expanding the home numeracy model to Chilean children: relations among parental expectations, attitudes, activities, and children's mathematical outcomes. *Early Childhood Research Quarterly*, *50*, 16-28.

*Swick, D. C. (2007). *The effects of parental involvement on children's school readiness skills* [Doctoral dissertation, University of North Carolina Chapel Hill].

*Thomson, D., Casey, B. M., Lombardi, C. M., & Nguyen, H. N. (2020). Quality of fathers' spatial concept support during block building predicts their daughters' early math skills–But not their sons'. *Early Childhood Research Quarterly*, *50*, 51-64.

*Thompson, R. J., Napoli, A. R., & Purpura, D. J. (2017). Age-related differences in the relation between the home numeracy environment and numeracy skills. *Infant and Child Development*, *26*(5), e2019.

Tracey, D. H., & Young, J. W. (2002). Mothers' helping behaviors during children's at-home oral-reading practice: Effects of children's reading ability, children's gender, and mothers' educational level. *Journal of Educational Psychology*, *94*(4), 729-737.

*Uscianowski, C., Almeda, M. V., & Ginsburg, H. P. (2020). Differences in the complexity of math and literacy questions parents pose during storybook reading. *Early Childhood Research Quarterly*, *50*, 40-50.

Van De Rijt, B. A., & Van Luit, J. E. (1999). Milestones in the development of infant numeracy. *Scandinavian Journal of Psychology*, *40*(1), 65-71.

*Vandermaas-Peeler, M., Boomgarden, E., Finn, L., & Pittard, C. (2012). Parental support of numeracy during a cooking activity with four-year-olds. *International Journal of Early Years Education*, *20*(1), 78-93.

Vandermaas-Peeler, M., Nelson, J., Bumpass, C., & Sassine, B. (2009). Numeracy-related exchanges in joint storybook reading and play. *International Journal of Early Years Education*, *17*(1), 67-84.

*Vandermaas-Peeler, M., & Pittard, C. (2014). Influences of social context on parent guidance and low-income preschoolers' independent and guided math performance. *Early Child Development and Care*, *184*(4), 500-521.

*Vasilyeva, M., Laski, E., Veraksa, A., Weber, L., & Bukhalenkova, D. (2018). Distinct pathways from parental beliefs and practices to children's numeric skills. *Journal of Cognition and Development*, *19*(4), 345-366.

Viechtbauer, W. (2010). Conducting meta-analyses in R with the metafor package. *Journal of Statistical Software*, *36*(3), 1-48.

*Vukovic, R. K., Roberts, S. O., & Green Wright, L. (2013). From parental involvement to children's mathematical performance: The role of mathematics anxiety. *Early Education & Development*, *24*(4), 446-467.

Vygotsky, L. S. (1978). Socio-cultural theory. *Mind in Society, 6*, 52-58.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London: Taylor & Frances/Routledge.

Wei, W., Li, Y., & Su, H. Y. (2020). Predicting the growth patterns in early mathematics achievement from cognitive and environmental factors among Chinese kindergarten children. *Learning and Individual Differences*, *79*, 101841.

Wolf, R., Morrison, J., Inns, A., Slavin, R., & Risman, K. (2020). Average effect sizes in developer-commissioned and independent evaluations. *Journal of Research on Educational Effectiveness*, *13*(2), 428-447.

Zhang, X., & Lin, D. (2017). Does growth rate in spatial ability matter in predicting early arithmetic competence?. *Learning and Instruction*, *49*, 232-241.

Zippert, E. L., & Ramani, G. B. (2017). Parents' estimations of preschoolers' number skills relate to at-home number-related activity engagement. *Infant and Child Development*, *26*(2), e1968.

*Zippert, E. L., & Rittle-Johnson, B. (2020). The home math environment: More than numeracy. *Early Childhood Research Quarterly*, *50*, 4-15.